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Application of Mathematica Software for Estimate the Fatigue Life Time Duration of Mechanical System

The paper present how we can use Mathematica to solve the equations types usually used to determinate the maximum stress cycles that can be support by a mechanical system until he will be out of use. To illustrate the type of equations used in specialized literature to estimate fatigue life time duration was chosen a specific case of mechanical structure applied to fatigue. It is about lever button of runner blade mechanism of Kaplan turbine, that in function support a very intensive alternative strain.

Keywords*: fatigue, equation, algorithm, mechanism*

1. Introduction

Lately, safety and reliability have become bywords to the mechanical industry at a time when energetically economy has dictated the use of the most sophisticated design procedures to optimize the use of material without increasing the probability of component failure. With the need to provide an economical design through mass reduction, many of the older design procedures have been set aside in lieu of the more recent and comprehensive design procedures.

Taking in consideration the extensive development of computer software package the time and costs of saffety designing are greatly reduced. The software packages not only calculate the flow of loads through components and the stresses at stress concentrations, but also are able to encompass large volumes and many channels of real time loading histories, and combine the two for fatigue life evaluations of complete mechanical systems [1].

The great disadvantage of that kind of software package is the costs and the technical limitation imposed by the huge data base necessary to be created for a most precisely results. Usually, the method used for estimate fatigue life duration presume determination of real stress in key point of a structure (the point with the greatest risk for crack) and than to determinate the maximum stress cycles that can be supported by the structure. The first part of the problem is resolved using the statically linear stress analyse (finite elements method). Currently, to find number of cycles until failure, is used Wőhler diagram of the analysed system material. To rise such kind of diagram supposes o lot of time and supplementary costs. Like an alternative, for solving the second part of the problem, a few researchers developed analytical equations that, even still disputed, are already applied.

2. Initial consideration

The two general equation models about us taking before are the Morrow and Smith-Watson-Topper (SWT) models [1]. The Morrow equation (1) modified the elastic term of the strain life equation for introducing the local mean stress into the strain life equation:

$$\frac{\Delta\varepsilon}{2} = \left(\frac{\sigma'_f - \sigma_m}{E}\right) \cdot \left(2N_f\right)^b + \varepsilon'_f \cdot \left(2N_f\right)^c \qquad (1)$$

where: $\Delta\varepsilon$ - total strain range;

- σ'_f - true fracture strength (value of σ_a at one reversal);
- σ_m - the mean stress;
- E – modulus of elasticity;
- ε'_f - fatigue ductility coefficient;
- N_f - number of cycles to failure; therefore $2N_f$ is equal to the number of reversals to failure;
- b - fatigue strength exponent;
- c - fatigue ductility exponent.

The SWT model assumes that by fatigue life for any condition of local mean stress depends on the product of $\left(\sigma_m + \sigma_a\right) \cdot \varepsilon_a$:

$$\left(\sigma_a + \sigma_m\right) \cdot \frac{\Delta\varepsilon}{2} = \left(\sigma'_f\right)^2 \cdot \left(2N_f\right)^{2b} + \sigma'_f \cdot \varepsilon'_f \cdot \left(2N_f\right)^{b+c} \qquad (2)$$

Both the Morrow and SWT approaches are in current use by the fatigue community and no consensus exists as which one is superior to other. Overall, the belief of the fatigue community is that the SWT approach provides good results and it is a good choice for general use. However, a review of current literature suggests that the Morrow result provides better results for compressive mean stresses while SWT gives non-conservative results.

To obtain more approach results from real exploitation conditions must calculate the number of cycle for every representative regime that occur in time on service. To select these regimes is necessary to have a good loads history usually obtained from monitoring process that always exists in modern industry.

After we have all the number of cycles the last problem is to apply a cumulative damage methodology for calculating the fatigue life. On these paper we will focus on the method used for solved such equation like the one propose by Morrow or SWT.

To resolve equation (1) will be used specialized software capable to obtain real answers from equations with a high degree of difficulty. It is about software named Mathematica a fully integrated software environment for technical and scientific computing. As we already said without such of software, actually, the problem can not be solved

Mathematica is one of the world's most respected software systems, and an essential tool for leaders in science and technology across the globe. Legendary for its sophisticated capabilities, yet easy enough to be used by children, *Mathematica* has considered as the most powerful general computation system ever created--and a complete computational environment for millions of people. Whether they have tasks that involve numbers, formulas, functions, graphics, data, documents, or interfaces, *Mathematica* gives automatic access to by far the largest collection of algorithms ever assembled[5].

2. Results and comments

The study system, used to exemplify how the software solve such equations is represented by lever's button from runner blade mechanism of a Kaplan turbines. Material of laver is 30CrNiMo8 and his mechanical properties are in accordance with current standards.

The work regime taking in considerations for this analyze are:

$H = 25\ m$ and $\varphi = 10^{\circ}$

With H is noted the head waterfall and φ represent the angular positions of runner blades. The equation choused, in accordance with specific exploitation conditions of the turbine, to typify the software algorithm are the Morrow relation (1).

The entry dates necessary to apply relation (1) are:

- characteristically dates for regime one necessary to calculate the number of stress cycles N_f [2] : - $\sigma_a = 262{,}3\ MPa$;

- $\sigma_m = 229{,}6\ MPa.$

- true fracture strength [4]: $\sigma'_f = 1230\ MPa$.

- fatigue ductility coefficient [4]: $\varepsilon'_f = 0{,}7$.

- total strain range [4]: $\Delta\varepsilon = 0{,}0025$.

- modulus of elasticity: $E = 205000\ MPa$.

The entry dates, except modulus of elasticity E, are calculate using measured dates in situ, respectively common formula proposed by specialized literature.

Having all the necessary entry dates it is obtained an equation having the following form:

$$0{,}00464 \cdot \left(N_f\right)^{-0{,}07} + 0{,}4309 \cdot \left(N_f\right)^{-0{,}7} - 0{,}00125 = 0$$

As seen, such of equation is practically impossible to solve without a specialized software. Here intervene the Mathematica software.

Built into *Mathematica* is the world's largest collection of both numerical and symbolic equation solving capabilities—with many original algorithms, all automatically accessed through a small number of exceptionally powerful functions. *Mathematica*'s symbolic architecture allows both equations and their solutions to be conveniently given in symbolic form, and immediately integrated into computations and visualizations.[3]

Solve — exact solutions to equations and systems
NSolve — general numerical solutions to equations and systems
FindRoot — numerically find local roots of equations

DSolve — exact solutions to differential equations
NDSolve — numerical solutions to differential equations

NSolve gives as a general way to find numerical approximations to the solutions of polynomial equations. Finding numerical solutions to more general equations, however, can be much more difficult, **FindRoot** gives as a way to search for a numerical root of a function or a numerical solution to an arbitrary equation, or set of equations.

The Mathematica sequence for finding the root of

$0{,}00464 \cdot \left(N_f\right)^{-0{,}07} + 0{,}4309 \cdot \left(N_f\right)^{-0{,}7} - 0{,}00125 = 0$ is in figure 1.

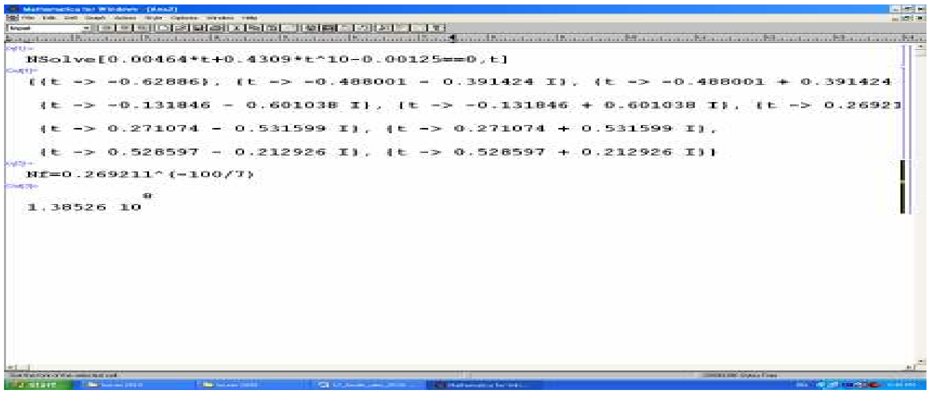

Figure 1.

The result of compilation with Mathematica software, for our specified system is:

$$N_f = 1{,}38 \times 10^8 \; cycles$$

Finally must be specified that the result obtained, in that form, it is know useful. That kind of equation can be applied only when we have mechanical system operating to constant operating loads. It is known, by practice, that such systems do not exist. For a real system is necessary to known the history of exploitation conditions, the made many such determination and then apply a cumulative damage methodology to find the final result.

4. Conclusions

The paper presents an alternative method that easier can be used to determinate de maximum number of stress cycles that can be support by a mechanical system, in known operating conditions.

The Mathematica software provides an inexpensive method whit minimal time allocated, to solve a very important problem of modern design concept. Only the design engineer is able to decide which method wants to apply.

The most part of engineers that already used the analytic method to solve real problem, considers that this alternative way is precisely enough to be used in designing mechanical systems.

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