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Zoltan Korka

# Dynamic Modelling of a Single Stage Cylindrical Gearbox

*Gearboxes are frequently used in machine systems for power transmission, speed variation and/or working direction. Dynamic modelling of gear vibration offers a better understanding of the vibration generation mechanisms as well as the dynamic behaviour of the gear transmission in the presence of gear tooth damage.*

***Keywords:*** *gearbox, vibration, gears, modelling, simulation*

## 1. Introduction

Because of their ubiquity and importance, gearboxes have received a considerable amount of attention regarding the dynamic modelling, a significant number of papers being published concerning this problem. Reference [1] is showing a short historical evolution of the most significant models used in the dynamic modelling of gears in mesh, respective gearboxes.

Vibration analysis has become a very important tool for detection of gear faults, and many signal processing procedures have been developed to extract information about incipient faults from the externally measured vibration signals.

The main objective of the present paper is to propose a dynamic model for a single stage cylindrical gearbox and to compare the model results obtained using a simulation program with the experimental data obtained from a test rig.

## 2. Modelling of the gear pair system

The mechanical model of the gear-pair system in mesh investigated in this paper is shown in Fig. 1. The gear mesh is modelled as a pair of rigid disks connected by a spring-damper set along the line of contact. Similar models were also considered in the references [5] and [6].

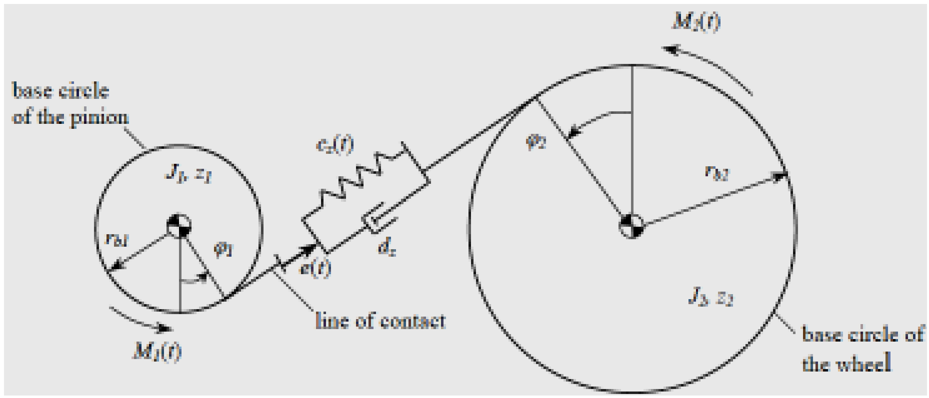


**Figure 1.** Mechanical model for the gear pair system

The model takes into account influences of the static transmission error which is simulated by a displacement excitation e(t) at the mesh. This transmissions error arises from several sources, such as tooth deflection under load, non-uniform tooth spacing, tooth profile errors caused by machining errors as well as pitting, scuffing of teeth flanks. The mesh stiffness $c_z$(t) is expressed as a time-varying function. The gear-pair is assumed to operate under high torque condition with zero back-lash. Effects of friction forces at the meshing interface are neglected on the basis that in particular, the coefficient of friction is low (approx. 6%). The viscous damping coefficient of the gear mesh $d_z$ is assumed to be constant.

The differential equations of motion for this system can be expressed in the form:

$$J_1\ddot{\varphi}_1 + r_{b1}c_z(t)[r_{b1}\varphi_1 + r_{b2}\varphi_2 + e(t)] + r_{b1}d_z[r_{b1}\dot{\varphi}_1 + r_{b2}\dot{\varphi}_2 + \dot{e}(t)] = M_1(t)\,, \quad (1)$$

$$J_2\ddot{\varphi}_2 + r_{b2}c_z(t)[r_{b1}\varphi_1 + r_{b2}\varphi_2 + e(t)] + r_{b2}d_z[r_{b1}\dot{\varphi}_1 + r_{b2}\dot{\varphi}_2 + \dot{e}(t)] = M_2(t)\,, \quad (2)$$

where $\varphi_i$ , $\dot{\varphi}_i$ , $\ddot{\varphi}_i$ (i= 1, 2 ) are rotation angle, angular velocity, angular acceleration of the input pinion and the output wheel respectively. $J_1$ and $J_2$ are the mass moments of inertia of the gears. $M_1$(t) and $M_2$(t) denote the external torques load applied on the system. $r_{b1}$ and $r_{b2}$ represent the base radii of the gears.

By introducing the composite coordinate

$$q = r_{b1}\varphi_1 + r_{b2}\varphi_2\,, \quad (3)$$

equations. (1), (2) yield a single differential equation in the following form:

$$m_{red}\ddot{q} + c_z(t)q + d_z\dot{q} = F(t) - c_z(t)e(t) - d_z\dot{e}(t)\,, \quad (4)$$

where: $m_{red} = \dfrac{J_1 J_2}{J_1 r_{b2}^2 + J_2 r_{b1}^2}$ , $\quad F(t) = m_{red}\left(\dfrac{M_1(t)r_{b1}}{J_1} + \dfrac{M_2(t)r_{b2}}{J_2}\right) \quad (5)$

For a specific gear-pair, the mesh stiffness $c_z(t)$ can be approximately represented by a truncated Fourier series:

$$c_z(t) = c_0 + \sum_{k=1}^{K} c_k \cos(k\omega_z t + \gamma_k), \qquad (6)$$

where $\omega_z$ is the gear meshing angular frequency which is equal to the number of gear teeth times the shaft angular frequency and $K$ is the number of terms of the series.

### 3. Experimental set-up

The experiment was done on the test rig described in reference [2] and shown in figure 3, using 4 pairs of gears (pinion & idler) whose teeth were machined with different helix angles: β= 9°, 11°, 13° and 15°.

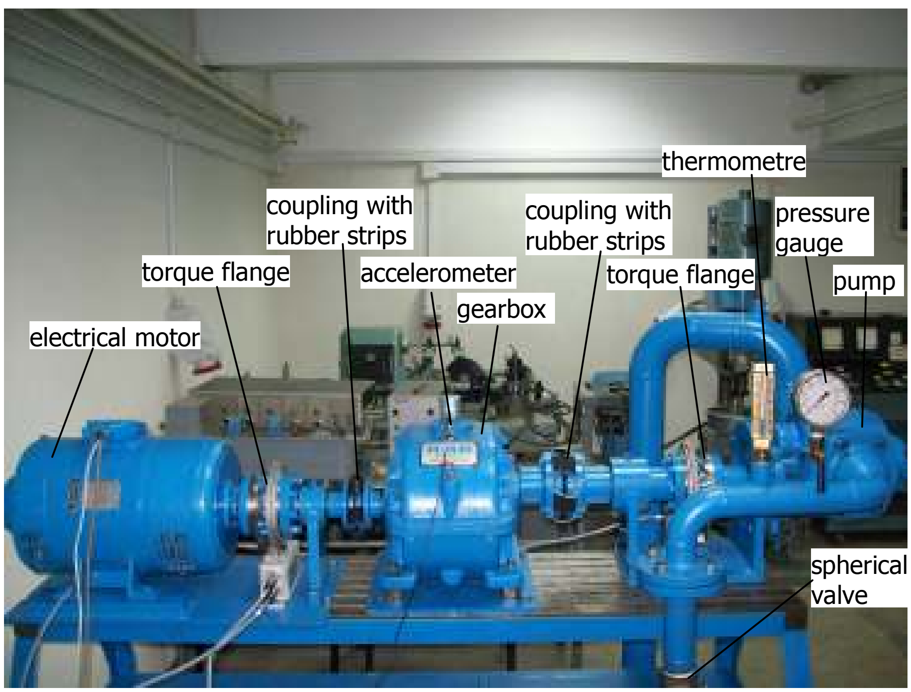


**Figure 3.** General view of the test rig

The main geometric data of the gears are presented in table 1. The mentioned data have following specification:

A- centre distance;
$m_n$ – module;
$z_1$, $z_1$– teeth number (1 for pinion, 2 for mating gear);
b- tooth width;
$\alpha_n$– normal pressure angle;
β- helix angle;
$x_1$, $x_2$-- addendum modification coefficients (1 for pinion, 2 for mating gear);
$d_{a1}$, $d_{a2}$-tip diameters of pinion (1), respective mating gear (2);
$d_1$, $d_2$- reference diameters of pinion (1), respective mating gear (2);
$d_{f1}$, $d_{f2}$- root diameters of pinion (1), respective mating gear (2).

**Table 1.**

| A= 125 mm<br>b= 50 mm | | $m_n$ = 4<br>$\alpha_n$= 20° | | $z_1$ / $z_2$ = 17/ 43 (i= 2,529)<br>back-lash: 0,15- 0,27 mm |
|---|---|---|---|---|
| | β= 9° | β= 11° | β = 13° | β= 15° |
| $x_1$/ $x_2$ | 0,48452/ 0,47656 | 0,39876/ 0,34228 | 0,30296/ 0,18148 | 0,20311/ -0,00731 |
| $d_{a1}$/ $d_{a2}$ | 80,04/ 185,28 | 80,04/ 185,54 | 80,02/ 185,79 | 79,99/ 185,98 |
| $d_1$/ $d_2$ | 68,85/ 174,14 | 69,27/ 175,22 | 69,79/ 176,52 | 70,40/ 178,07 |
| $d_{f1}$/ $d_{f2}$ | 61,73/ 166,90 | 61,47/ 166,90 | 61,22/ 166,92 | 61,03/ 166,95 |

For vibration measurements, there were used Bruell & Kjaer accelerometers of type 4524B, mounted on special prepared surfaces on the housing, as described in reference [3] and shown in Figure 4.

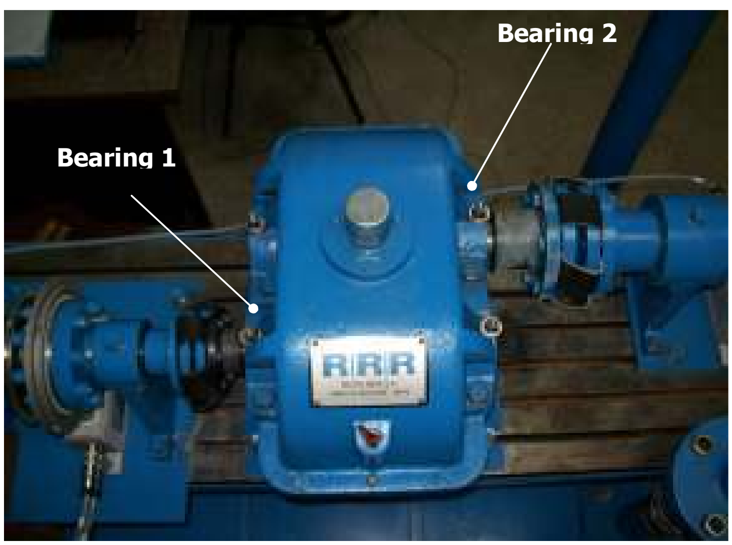


**Figure 4.** Vibration measuring points on the gearbox housing

### 4. Numerical simulation and experimental comparisons

Computer simulation was done using the simulation program SimulationX3-Student Edition, developed by ITI GmbH Dresden, made available, without charge, for a limited time, to the article's author.

Dynamic modelling of the gear transmission was started from the model described in chapter 2. Using the library provided by the simulation program, it was built a model as shown in Figure 5.

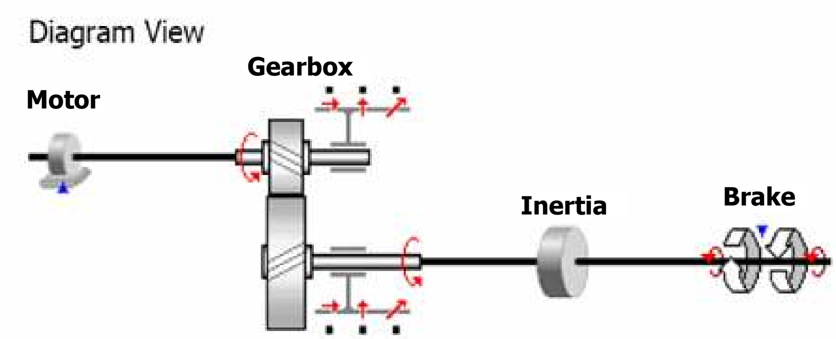


**Figure 5.** Dynamic model for numerical simulation of the cylindrical gearbox

Using the above defined model there were run simulation calculations for the four pairs of gears with teeth inclination of 9°, 11°, 13° and 15° at the nominal speed (1.500 rpm) of the electrical motor.

The curves of variation of normal force on the tooth flank, calculated for the four gear pairs are shown in Figures 6 to 9. Obviously, as shown in figures, the normal force on tooth has a variation with a period T of approx. 0.00235 s, which corresponds to a frequency f= 1/ T= 1/ 0,00235= 425 Hz equal to the gears frequency fz, calculated at the motor speed.

Hence, it can be concluded that the variation in time of the normal force on the tooth can be treated as the vibration generated by the gear pairs.

From the Figures 6 to 9 it can be measured the amplitudes (peak - peak) of the force variation, results being presented in Table 2.

**Table 2.**

| Helix angle β | 9° | 11° | 13° | 15° |
|---|---|---|---|---|
| Maximum normal force [N] | 740 | 655 | 625 | 610 |
| Minim normal force [N] | 250 | 340 | 340 | 380 |
| Amplitudine peak- peak[N] | 490 | 315 | 285 | 230 |
| Percentage variation of normal force | 100% | 64,3% | 58,2% | 46,9% |

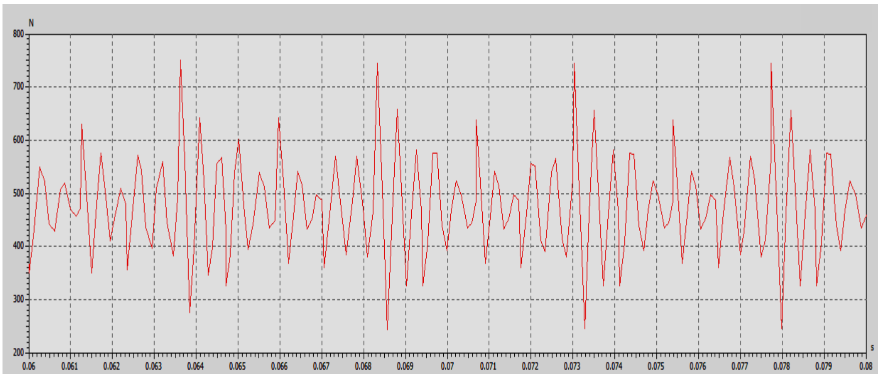


**Figure 6.** Normal force variation on the gear pair with β=9°


**Figure 7.** Normal force variation on the gear pair with β=11°

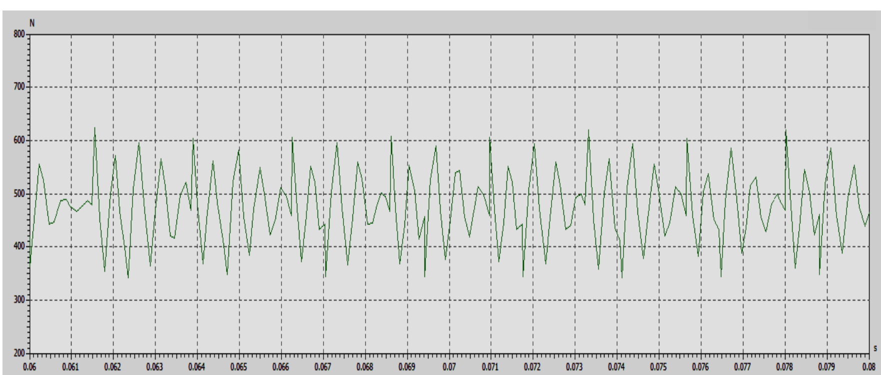


**Figure 8.** Normal force variation on the gear pair with β=13°

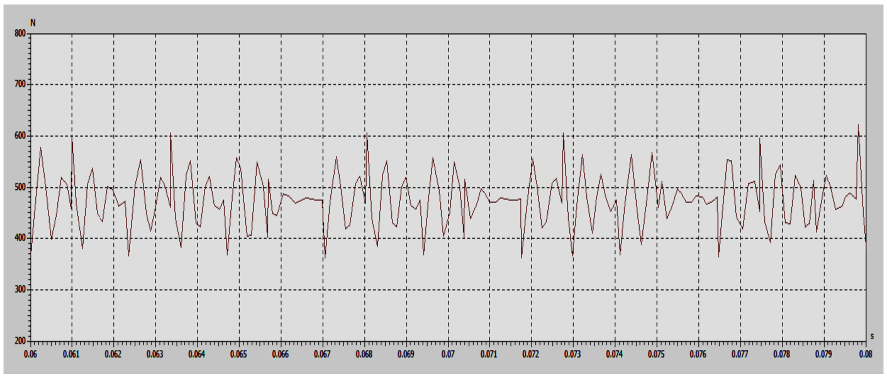

**Figure 9.** Normal force variation on the gear pair with β=15°

Based on the above mentioned figures, it can be concluded that increasing the inclination of teeth is decreasing the gear vibration amplitude, fact which is also confirmed by the measurements performed on the test rig. Consequentially, the mechanical model proposed for the gearbox in Figure 1 can be validated.

The experimental measurement accuracy as well as the validity of the mechanical model chosen for the dynamic simulation of the gear transmission are also confirmed by Figure 10. The chart is showing on one hand the percentage change of the maximum vibration velocities measured in the radial plane of one of the bearings and, on other hand, the percentage change in amplitude of the normal force on the tooth determined by the numerical simulation.

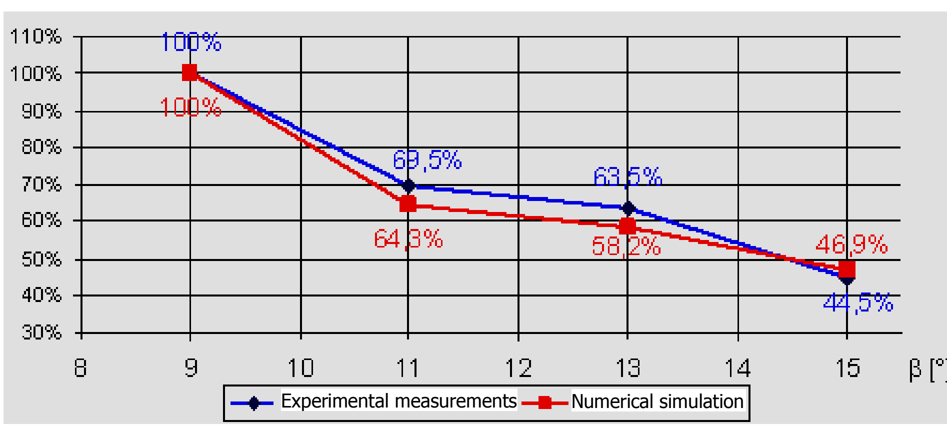


**Figure 10.** Percentage change in of maximum vibration amplitudes

## 5. Conclusions

In the above sections, the parametrically excited vibration of a gearbox was investigated. A comparison between the model result and experimental data result was also presented. The target of the study was to provide the fundamental understanding of the physical mechanism related to the gear vibration.

Here it must be noted that although the mechanical model is a relative simple one, but it can be able to reveal essential dynamic properties of the gear pair in mesh. No attempt is made here to present a generous study on the effects of external load variation, variations of the mesh stiffness caused by local tooth faults, etc., and a mathematical treatment of this problem is let for future investi-gation. However, the obtained results seem promising and extension to more complicated geared systems.

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*Address:*

- Phd. Eng. Zoltan KORKA, CEO Resita Reductoare S.A., Platforma Calnicel, 320151, Reşiţa, zkorka@resitareductoare.com