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Model Equations of Shape Memory Effect - Nitinol

Even it has been already confirmed that SMA's have high potential for robotic actuators, actuators included in space robotics, underwater robotics, robotics for logistics, safety, as well as "green robotics" (robotics for the environment, energy conservation, sustainable development or agriculture), the number of applications of SMA-based actuators is still quite small, especially in applications in which their large strains, high specific work output and structural integration potential are useful,. The paper presents a formulated mathematical model calculated for binary SMA (Ni-Ti), helpful to estimate the stress distribution along with the transformation ratio of a SMA active element.

Keywords: *shape memory, modeling, actuation, thermo mechanical transfer, Nitinol*

1. Introduction

A solution of a SMA robotic actuator induces the development of an innovative solution that is not only economically (energy efficient, cost savings) but ecologically sustainable. This fact due to the necessity of a deeper understanding of the thermo-mechanical behavior of SMA and how it might be exploited in the design of working actuators is necessary, involving the necessity to develop a mathematical model able to describe all thermo-mechanical properties of SMA by relatively simple final set of constitutive equations.

Shape memory effect (SME) can be observed to a limited number of alloys, being characterized by recovery of original shape on heating above certain temperature after significant deformation (up to 6%) of the shape memory material. Regarding this SME, during time have been distinguished three phenomena: one-way shape memory effect, two-way shape memory effect and pseudo-elasticity (known also as superelasticity). Both one-way SME and two-way SME comprise recovery of deformed SMA upon heating. The basic difference between one-way and two-way SME is that no reverse change of SMA's shape occurs in case of one-way SME after subsequent cooling whereas two-way SME is characterized by a change of SMA's shape during cooling. Two-way SME exhibits switching between ''cold'' shape and ''hot'' shape upon cooling down and heating up respectively.

The loading path differs from unloading path in the stroke-temperature diagram. Both paths are parts of a hysteresis loop.

For practical applications of SMA device such as actuators or thermo-sensitive devices, it is indispensable to design and estimate basic performance qualitatively.

2. Mathematical Model.

On the base of our previous analysis, Ni-substructure has behaviour of elasto-plastic material whereas Ti- substructure has behaviour of elastic material. These conditions are reflected in the constitutive equations (3a), (5a), (6a) and (8a) assumed for Ni- and Ti- substructure. The constitutive equations express a relationship among micro-stresses, micro-strains in Ti- substructure (index t attached to the corresponding tensors), micro-stresses and micro-strains in Ni- substructure (index n attached) and macro-stresses and macro-strains (marked by overbar). Volume fractions of Ni and Ti constituents were introduced and denoted as v_n, v_t respectively $(v_n = 1 \quad v_t)$. We suppose validity of the following equations:

$$v_n \sigma_{ijn} + v_t \sigma_{ijt} = \bar{\sigma}_{ij}\,, \tag{1}$$

$$v_n \varepsilon_{ijn} + v_t \varepsilon_{ijt} = \bar{\varepsilon}_{ij}\,, \tag{2}$$

where $\varepsilon_{ijn}, \varepsilon_{ijt}$ are deformation tensors associated with Ni- and Ti- substructures respectively, $\sigma_{ijn}, \sigma_{ijt}$ are stress tensors associated with Ni- and Ti- substructure respectively, $\bar{\sigma}_{ij}$. and $\bar{\varepsilon}_{ij}$ are tensors of macroscopic stress and strain respectively.

The set of constitutive equations designed for a material consisting of two material constituents was adapted for binary SMA as follows:

$$de_{ijn} = \mu ds_{ijn} + s_{ijn} d\lambda_n\,, \qquad \varepsilon_n = \rho\sigma_n = \rho\bar{\sigma} = \bar{\varepsilon}\,, \tag{3}$$

$$de'_{ijn} = de_{ijn} - d\bar{e}_{ij}\,, \qquad \varepsilon'_n = \varepsilon_n - \bar{\varepsilon}\,, \tag{4}$$

$$de'_{ijn} = \mu ds'_{ijn} + s'_{ijn} d\lambda_n\,, \qquad \varepsilon'_n = 0\,, \tag{5}$$

$$de_{ijt} = \mu ds_{ijt}\,, \qquad \varepsilon_t = \rho\sigma_t = \rho\bar{\sigma} = \bar{\varepsilon}\,, \tag{6}$$

$$de'_{ijt} = de_{ijt} - d\bar{e}_{ij}\,, \qquad \varepsilon'_t = \varepsilon_t - \bar{\varepsilon}\,, \tag{7}$$

$$de'_{ijt} = \mu ds'_{ijt}\,, \qquad \varepsilon'_t = 0\,, \tag{8}$$

where e_{ijn}/s_{ijn} are the deviatoric parts of the average strain/stress tensors $\varepsilon_{ijn} / \sigma_{ijn}$ in the n - material constituent, e_{ijt} / s_{ijt} are the deviatoric parts of the average strain/stress tensors $\varepsilon_{ijt} / \sigma_{ijt}$ in the t - material constituent, $\delta_{ij}\varepsilon_n / \delta_{ij}\sigma_n$ are isotropic parts of the average strain/stress tensors $\varepsilon_{ijn} / \sigma_{ijn}$ in the n - material constituent, $\delta_{ij}\varepsilon_t / \delta_{ij}\sigma_t$ are isotropic parts of the average strain/stress tensors $\varepsilon_{ijt} / \sigma_{ijt}$ in the t - material constituent. The macroscopic values of tensors are indicated again by overbars. The symbols $\mu[= (1+v)/E]$ and $\rho[= (1 \quad 2v)/E]$ mean elastic compliances, v meaning Poisson's ratio and E Young's modulus. The symbols with primes - defined by equations (4) and (7) - characterize the influence of the heterogeneity of strain- and stress- fields. The symbol λ_n stands for scalar measure of inelastic deformation in n-material constituent. Equations (6a) and (8a) express elastic character of Ti-substructure whereas equations (3a) and (5a) describe elasto-plastic behaviour of Ni-substructure. We assume validity of Misses' criterion for elasto-plastic deformation in Ni-substructure.

Equations (1) and (2) are also valid for the deviatoric parts of stress and strain tensors:

$$v_n s_{ijn} + v_t s_{ijt} = \bar{s}_{ij}\,, \tag{9}$$

$$v_n e_{ijn} + v_t e_{ijt} = \bar{e}_{ij}\,, \tag{10}$$

Thus, we have nine unknown tensorial differentials: ds_n, ds_t, de_n, de_t, $d\bar{e}$, $ds_n{}'$, $ds_t{}'$, $de_n{}'$ and $de_t{}'$. In order to determine them, we have only 8 equations available: (3) - (8), (9) and (10). Therefore, a new equation is necessary to add. This new equation was derived in the quoted monograph on condition taking only mechanical loading into account and has the form:

$$(s_{ijn})_M - (s_{ijt})_M + \frac{s'_{ijn}}{\eta_n} - \frac{s'_{ijt}}{\eta_t} = 0\,, \tag{11}$$

where η_n, η_t are the so called "structural parameters". These structural parameters are supposed to be constant for diffusionless processes where atomic neighbours remain unchanged. They can be determined on the basis of fitting the computed and the experimental stress-strain curve. Index M indicates that only effect of mechanical loading is taken into account. The total stresses consist of stresses due to mechanical loading and stresses due to temperature changes and are defined as follows:

$$\sigma_{ijn} = (\sigma_{ijn})_M + (\sigma_{ijn})_T, \sigma_{ijt} = (\sigma_{ijt})_M + (\sigma_{ijt})_T\,, \tag{12}$$

where $(\sigma_{ijt})_M$ is stress tensor due to mechanical loading and $(\sigma_{ijt})_T$ is stress tensor due to temperature changes. In our approach, the effect of temperature changes is connected with the differences in thermal expansions of the two elements present in the binary alloy. Temperature changes and the difference in thermal dilatation of the two substructures result in self-equilibrated stresses which can be expressed mathematically:

$$\sigma_{ijn} = (\sigma_{ijn})_M + (\sigma_{ijn})_T , \sigma_{ijt} = (\sigma_{ijt})_M + (\sigma_{ijt})_T , \quad (13)$$

Equations (14) and (15) are valid for their deviatoric parts of stress tensors as well:

$$s_{ijn} = (s_{ijn})_M + (s_{ijn})_T , s_{ijt} = (s_{ijt})_M + (s_{ijt})_T , \quad (14)$$

$$v_n (s_{ijn})_T + v_t (s_{ijt})_T = 0 , \quad (15)$$

For the purpose of our set of equations is solvable we need to express tensorial variable $(s_{ijn})_T$ or tensorial variable $(s_{ijt})_T$ by means of the other tensorial variables. According to our concept, the SME appears after inelastic deformation of Ni-substructure.

This inelastic deformation is recovered during heating and the process of the recovery is directly connected with a change of $(s_{ijn})_T$. Supposing that $(s_{ijn})_T$ depends on the deviatoric measure of deformation of Ti- substructure that is caused by inelastic deformation of Ni- substructure, it is easy to see that $s_{ijt}{}'$ has such property: In the elastic range near to a stress-free virgin state it has zero value, it is not changed by any purely elastic process (e.g. by elastic unloading), it changes only in the course of inelastic deformation of the Ni-substructure. This can be seen from equations (8), (7) and (2).

$$[d s'_{ijt}]_{el} = \frac{1}{\mu}[d e'_{ijt}]_{el} = \frac{1}{\mu}[de_{ijt}]_{el} - [d \bar{e}_{ij}]_{el} = 0 , \quad (16)$$

Our objective is to find a relationship between recovery strain and temperature. For introduction of the thermal effects it is convenient to start from some specific temperature that is less than A_s, has no influence on the recovery process. This should be temperature To corresponding to fully martensitic state of the material. One of our assumptions is that the state of the material prior to SME is fully martensitic and the temperature of this state is identified with T_o. Deviation from this temperature is defined as

$$\tau = T - T_o , \quad (17)$$

Supposing that $(s_{ijn})_T$ depends on $s_{ijt}{}'$ and on τ. The simplest formula that takes into account both of the dependencies is following:

$$(s_{ijn})_T = \omega \tau s'_{ijt} , \quad (18)$$

or - in a differential form:

$$(ds_{ijn})_T = \omega \tau ds'_{ijt} + \omega s'_{ijt} d\tau , \quad (19)$$

where ω is constant for processes of the same character. Because our model is supposed to deal with physically different processes of heating and cooling with and without inelastic deformation ω can take on different values.

From the above set of equations the macroscopic constitutive equation with the respective evolution equations for internal variables is derived in the following form:

$$d\overline{e}_{ij} = \mu d\overline{s}_{ij} + v_n s_{ijn} d\lambda_n , d\overline{\varepsilon} = \rho d\overline{\sigma} \,, \tag{20}$$

$$ds_{ijn} = d\overline{s}_{ij} \quad \frac{1}{P + R + \omega\tau R v_n / v_t}\{[(v_t P + \omega\tau\eta_n \eta_t v_n) s_{ijn} \quad , \tag{21}$$

$$v_t \eta_t s'_{ijn}]d\lambda_n / \mu \quad \omega\eta_n \eta_t s'_{ijt} d\tau\tau$$

$$ds'_{ijn} = ds_{ijn} \quad d\overline{s}_{ij} + (v_t s_{ijn} \quad s'_{ijn}) d\lambda_n / \mu \,, \tag{22}$$

$$ds'_{ijt} = v_n \quad \frac{d\overline{s}_{ij} \quad ds_{ijn}}{v_t} \quad s_{ijn} \frac{d\lambda_n}{\mu} \quad , \tag{23}$$

where

$$P = v_n \eta_n + v_t \eta_t , R = \eta_n \eta_t \,, \tag{24}$$

are constant to be determined.

$$d\hat{\lambda}_n = \mu \frac{(P + R + \omega\tau R v_n / v_t) s_{ijn} d\overline{s}_{ij} + R\omega\omega_{ijn} s'_{ijt} d\tau}{(v_t P + v_n R\omega\omega\tau)_{ijn} s_{ijn} - v_t \eta_t s_{ijn} s'_{ijn}} \,, \tag{25}$$

The relationship between $d\lambda_n$ and $d\hat{\lambda}_n$ is defined by equation (10).

The structural parameters η_n , η_t volume fractions of Ni and Ti constituents v_n , v_t , elastic compliances μ and ρ , elastic limit c_n and ω are parameters and constants to be determined. They are constant for a SMA specimen in question. Their determination is a relatively complex problem and is based on fitting the computed and the experimental stress-strain curve of a SMA specimen.

4. Conclusion

The growing demand for technological innovation to enable empowerment of developing communities requires good understanding of the SMA's behavior in order to produce actuators suitable for robots both autonomous and cooperative, to enable robots to accomplish very delicate tasks for industrial, educational or biomedical applications. The paper try to introduce a model of thermo-mechanical behaviour of Nitinol, considering separately mechanical properties of Ni- and Ti- substructure. The mechanical behaviour of Ti- substructure is modelled as elastic whereas that of Ni- substructure as elasto-plastic. The model outputs are relatively simple differential constitutive equations.

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