ANALELE UNIVERSITĂȚII
“EFTIMIE MURGU” REȘIȚA
ANUL XVII, NR. 1, 2010, ISSN 1453 - 7397

Ovidiu Vasile

Transmission Loss Assessment for a Muffler by Boundary Element Method Approach

This paper investigates the acoustic performance of two cases for reactive mufflers using Boundary Element Method (BEM) analysis. Modeling procedures for accurate performance prediction had led to the development of new methods for practical muffler components in design. The transmission loss (TL) is the more widely can be easily computed with a BEM analysis. The author presents an overview of the principles and theoretical formulation of BEM for predicting the transmission loss of a muffler, the pressure and velocity distribution on surfaces of muffler. At the end of the paper is presented a comparison of two cases of mufflers for transmission loss. The predicted results agreed in some limits with the experimental data published in literature.

***Keywords**: muffler, transmission loss, BEM*

1. Muffler performances

The most widely used performance used to characterize mufflers is surely the transmission loss (TL), other indexes are however available such as insertion loss (IL) and noise reduction (NR), and a good understanding of the differences among them is fundamental in order to apply the most appropriate to each situation. Considering a generic muffler or duct as depicted in Figure 1, we have that the pressure p_1 at the inlet is composed by two waves one traveling towards right (entering the muffler) that is called p_1^+ , and the other traveling in the opposite direction and called p_1^-.

At the outlet the situation is similar and the total pressure p_2 is composed by two waves traveling in opposite directions. The velocity at the inlet (V_1) and outlet (V_2) sections can also be expressed in terms of the two components of the waves.

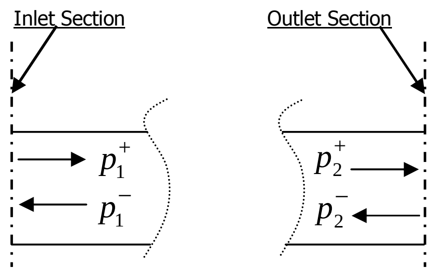


Figure 1. The inlet and outlet of muffler or duct.

The overall relations are [1,3,4,5]:

$$p_1 = p_1^+ + p_1^- \tag{1}$$

$$V_1 = (1/\rho_0 \cdot c) \cdot \left(p_1^+ - p_1^-\right) \tag{2}$$

$$p_2 = p_2^+ + p_2^- \tag{3}$$

$$V_2 = (1/\rho_0 \cdot c) \cdot \left(p_2^+ - p_2^-\right) \tag{4}$$

where ρ_0 - is the air density

The *TL* is defined as the ratio between the sound power that actually enter in the muffler and the transmitted sound power. The sound power that enters in the muffler is associated to the right traveling wave at the inlet (p_1^+), while the transmitted sound power is associated to the right traveling wave at the outlet (p_2^+). In other words the *TL* is the ratio $(p_1^+)^2/(p_2^+)^2$.

The transmission loss is the more widely used mainly because it can be more easily evaluated theoretically since it is an intrinsic property of the muffler, while the Insertion loss depends instead of the acoustic impedance at the inlet and outlet. If the impedance at inlet and outlet are both equal to the fluid impedance, then the insertion loss is equal to the transmission loss.

2. Evaluation of the transmission loss TL

The standard procedure for evaluation of TL is based on the evaluation of the so-called four pole parameters (A, B, C, D) that characterize the muffler. In the past several studies were conducted in order to analytically evaluate these parameters, but nowadays they can be easily computed with a BEM analysis. It is simply required to execute two set of calculations that differs only for the boundary conditions applied at the outlet. The calculations to be performed are respect the Table 1 [2,3,4]:

Table 1. Boundary conditions

Set	Boundary condition at inlet	Boundary condition at outlet
1	Imposed velocity v=1	Imposed velocity v=0
2	Imposed velocity v=1	Imposed pressure p=0

The four parameters (that are complex numbers that depends on frequency) can then be computed as [2,3,5]:

$$A=\left(p_1/p_2\right);\; C=\left(v_1/p_2\right) \text{ from set 1} \tag{5}$$

$$B=\left(p_1/v_2\right);\; D=\left(v_1/v_2\right) \text{ from set 2} \tag{6}$$

An interesting properties of the above parameters is that they satisfy the relation AD-BC=1, and this can be used as an useful check for ensuring the accuracy of the performed calculations. Using the equations (1), (2), (3), (4) and the above definitions of A, B, C, D it is possible to obtain an expression for the TL.

The transmitted pressure p_2^+ can be most easily determined if the outlet is non-reflecting that is if p_2^- =0. Using then equations (1), (2), (3), (4) and the above definitions the ration $(p_1^+)/(p_2^+)$ can be easily obtained and the transmission loss writes as [2,3,4,5]:

$$TL=20\log_{10}\left[\left|A+\frac{B}{\rho\cdot c}+C(\rho\cdot c)+D\right|\Big/2\right] \tag{7}$$

3. BEM acoustic theoretical formulation

The basic equation for acoustic wave propagation through an elastic medium is the linear wave equation [3]:

$$\nabla^2 u=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}+b \tag{8}$$

where $u(x,t)$ is the velocity potential, c is the speed of sound, $b(x,t)$ is the sound source, x and t are the position and time variables. Assuming that the problem is time harmonic, equation (8) can be transferred to the frequency domain so as to obtain the Helmholtz equation [3,5]:

$$\nabla^2 u+k^2 u=b \tag{9}$$

where $k=\omega/c$ is the wave number and ω the angular frequency. Using the concept of free field Green's function $\left(v^*,u^*\right)$, the Helmholtz equation can be converted in the following integral equation, defined on the boundary[3,5]:

$$c(P)u(P)+\int_S v^* u dS = \int_S u^* v dS \tag{10}$$

Equation (10) can be expressed in a boundary element formulation, in order to aplly a numerical resolution method (in most cases the analytical treatment is overwhelmingly difficult)

$$c(P)u(P)+\sum_{N_{elements}} \int_{S_{element}} v^* u dS = \sum_{N_{elements}} \int_{S_{element}} u^* v dS \tag{11}$$

where $c(P)$ is dependent on the domain geometry, v is the fluid particle velocity and S hte boundary surface. By substituting in equation (11)

$$u(x) = -\,p(x)/i\omega\rho \tag{12}$$

where ρ is the mass density of the acoustic media, it is possible to write equation (11) in matrix form:

$$\mathbf{HP} = \mathbf{GV} + \mathbf{B} \tag{13}$$

where $\mathbf{P}$ and $\mathbf{V}$ are vectors of nodal pressures and velocities on the BEM surface, while $\mathbf{B}$ is a body source vector. For a given velocity field on the panel, an acoustic BEM direct frequency response analysis calculates and stores the following data in the model database: pressure and velocities values in nodes on the BEM surface and at field points. It is worth to point out that the matrices $\mathbf{H}$, $\mathbf{G}$ are fully populated, involving long run times for the system resolution. The pressure at an arbitrary field point is obtained by postprocessing surface pressure and normal velocity values: in this case only numerical integration is needed. There is one row and column for each boundary element node in the model and the matrices $\mathbf{H}$ and $\mathbf{G}$ are frequencies dependent so as to require a full acoustic analysis for each frequency of interest.

If the fluid is not supposed to be conservative its physical properties are complex and consequently the solution is complex, existing phase relationships between the physical quantities like pressure and velocity, but this is not the case for our problem where an ideal fluid is considered.

A critical issue for an accurate evaluation of *TL* and *IL* is the correct application of Boundary Conditions (in the following abbreviated as BC), in particular in regions where they change.

For the inlet region, we need to apply a constant velocity at the inlet section while in the other nodes of the duct the BC is still an imposed velocity but with zero velocity [2]. The situation is depicted in the Figure 2.

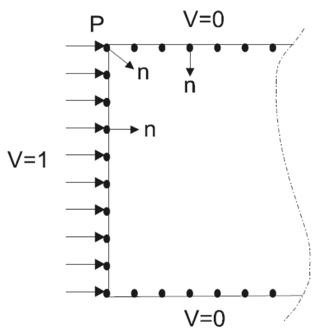


Figure 2. Inlet region

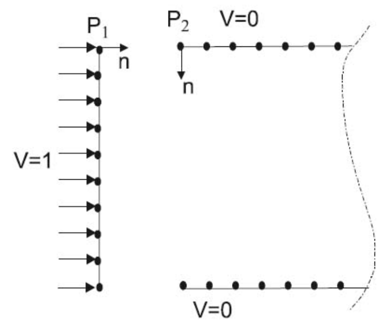


Figure 3. Operation of splitting the nodes

Consider now the point P that is at the intersection of the inlet section with the duct surface. Velocity has to be applied to this point, if we consider this point as belonging to the inlet section we should impose a unitary normal velocity while if we consider it as belonging to the duct we should impose a null velocity. The correct velocity to apply, if we come back to the definition of the velocity BC we remember that this BC consist in ensuring that the fluid velocity in the direction of the perpendicular to the surface be equal to the imposed value. But what is the direction of the surface normal for the point P. Theoretically speaking the normal is not defined since the surface is not smooth at this point, however practically the surface normal for a generic point is always computed taking the average of the normal of all the panels at which the node is connected.

The right solution is the possibility to split the node P in two nodes P_1 and P_2 having the same geometrical coordinates but one connected to the panel of the inlet section and the other connected to the panel of the duct, as depicted in Figure 3. In the picture the view is exploded and the points P_1 and P_2 are showed at different places but this is only for visualization reason and they should have instead the same geometrical coordinates. The important thing is however that the node P1 be connected only to the panel of the inlet section and the node P_2 be connected only to the panel of the duct. Now the surface normal for the point P_1 is horizontal since the point is no more connected to any panel of the duct. Reciprocally the normal of the point P_2 is now vertical. This operation of splitting the nodes is referred as disconnection, since the elements of the inlet are no more topologically connected to the elements of the duct [2].

The same kind of problem can appear for example at the outlet. In the point at intersection of outlet and duct, this situation is also more difficult since we have that different kind of BC should be applied to points of duct and outlet, since for the outlet section we need to assign a pressure BC while for the duct we have as usual a Velocity BC. The correct BC to be applied for the point at intersection can now be easily obtained, also in this case we need to introduce two coincident nodes, one connected to the panels of duct and the other connected to the panel of outlet and apply the relative BC to each node.

4. Muffler analysis and results

As a practical example we are now going to consider two cases of muffler. *Case 1* for a simple expansion chamber and *Case 2* for two expansion chamber[2,3]; then using the VNoise software we are going to evaluate the transmission loss *TL* and finally compare them with each.

The base models of the two cases are defined inserting the nodes that define the profile of the muffler and then is generate a revolution surface from them. The nodes coordinates to be inserted are presented in Table 2 and Table 3, and then connecting them with edges we obtain the base model represented in both cases Figure 4 and Figure 5.

The two cases differ only because of introduction or deflector panel (N_5N_6 in Figure 5), thus dividing the inner chamber into two chambers that communicate with each other by a certain circular section.

Table 2. The nodes coordinates for case 1

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
x	-0,4	-0,4	-0,3	-0,3	0,3	0,3	0,4	0,4
y	0	0,05	0,05	0,2	0,2	0,05	0,05	0
z	0	0	0	0	0	0	0	0

Table 3. The node coordinates for case 2

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}
x	-0,40	-0,40	-0,30	-0,30	0	0	0,30	0,30	0,40	0,40
y	0	0,05	0,05	0,20	0,20	0,05	0,20	0,05	0,05	0
z	0	0	0	0	0	0	0	0	0	0

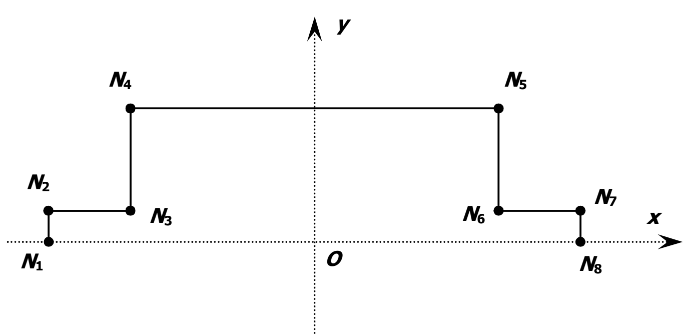


Figure 4. Base model of muffler for case 1 (N_1 … N_8)

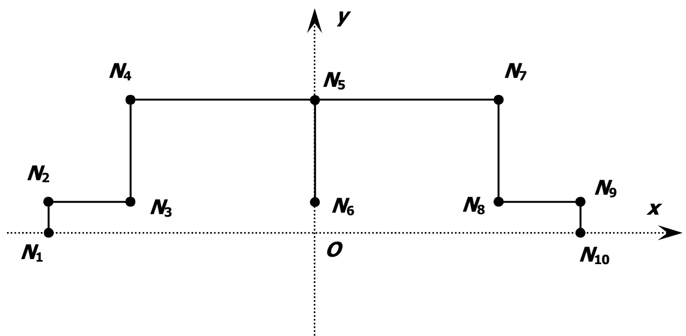
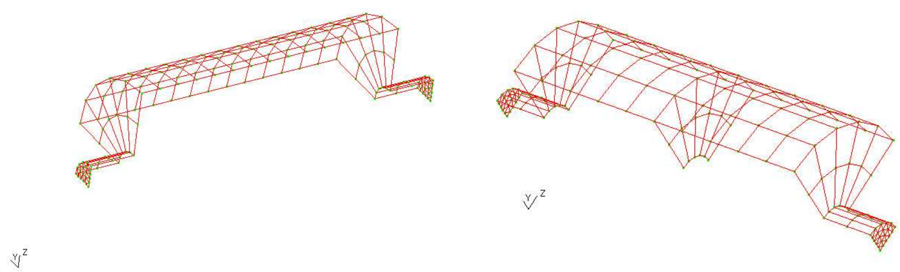


Figure 5. Base model of muffler for case 2 (N_1 … N_{10})

We apply the required BC and then perform the discretization using 6 points per wave at 4500 Hz. Figure 6 show discretization of muffler, for example to generate a 90° revolution surface and use symmetries during calculations.



a. b.

Figure 6. Discretization of muffler:
a) *case 1* – one expansion chamber; b) *case 2* – two expansion chamber

In Figures 7 - 10 is show velocity distribution and pressure distribution on surfaces of muffler with single expansion chamber and for a muffler with two expansion chamber for 3000 Hz. In the second case are presented in Figures 11 and 12 is show velocity distribution and pressure distribution on surfaces of inside baffle plane with two expansion chamber for 3000 Hz

In order to evaluate the transmission loss (*TL)* we need to execute two set of calculation, one with v=0 at the outlet and the other with p=0 at the outlet (see Table 1).

First of evaluating the TL it is a good practice to check that convergence is achieved. In muffler analysis a good method to check convergence is to check the values of [1-(AD-BC)]. In our example the values of [1-(AD-BC)] are much smaller then unity [3] and therefore it can be a good indication that convergence is achieved.

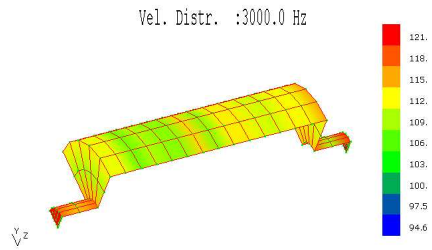


Figure 7. Velocity distribution on surfaces of muffler at 3000 Hz for *case 1*-

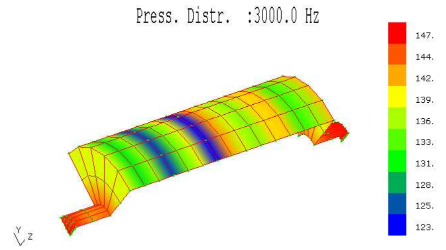


Figure 8. Pressure distribution on surfaces of muffler at 3000 Hz for case 1

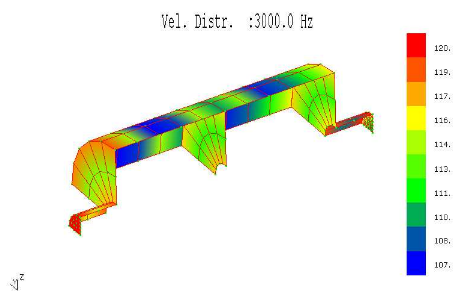


Figure 9. Velocity distribution on surfaces of muffler at 3000 Hz for case 2

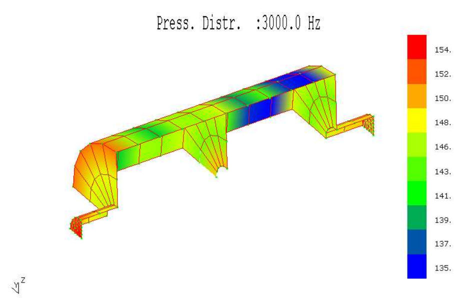


Figure 10. Pressure distribution on surfaces of muffler at 3000 Hz for case 2

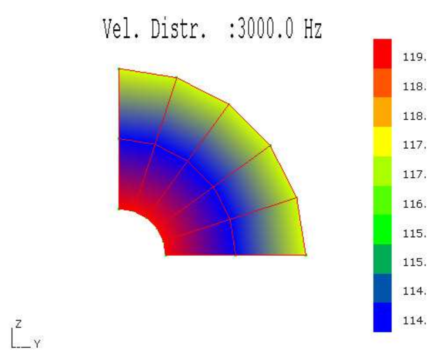


Figure 11. Velocity distribution on surfaces of baffle plane at 3 kHz for case 2

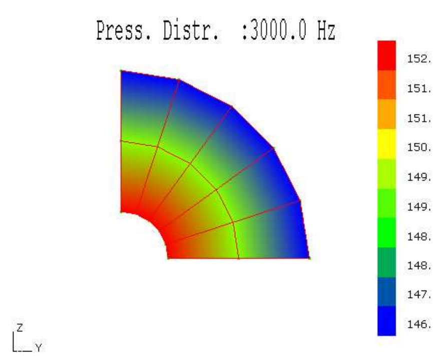


Figure 12. Pressure distribution on surfaces of baffle plane at 3 kHz for case 2

5. Conclusion

We consider a calculation in the range of (50-3300 Hz) with a step of 10 Hz, using a rotational symmetry, consider only a ¼ of the muffler. In this case on Figure 13 is presented transmission loss (*TL*) for muffler for both cases.

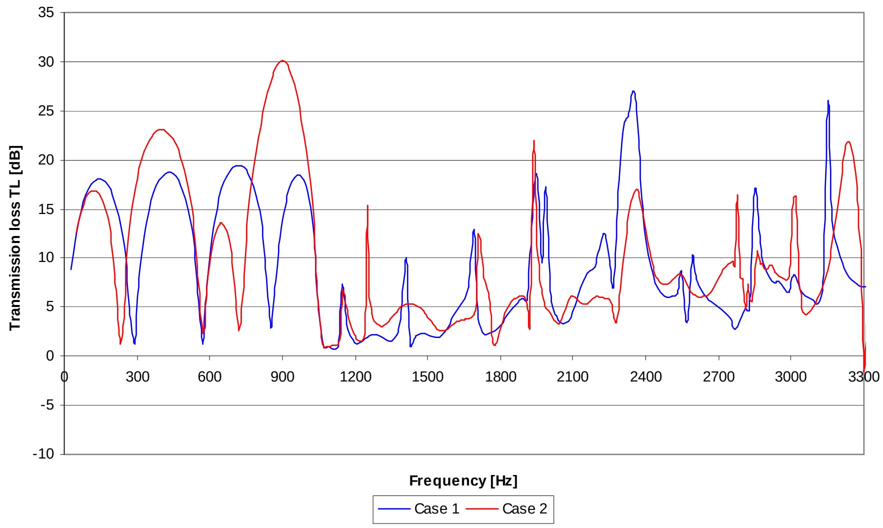


Figure 13. Transmission loss (*TL*) for muffler with:
Case 1 - one expansion chamber; Case 2 - two expansion chamber

Once that we provide the two set of calculations, the two nodes respectively of inlet and outlet to be used for *TL* evaluation, then will may automatically evaluate the four parameter A,B,C,D and transmission loss (TL) are given in Figure 13. The predicted results agreed in some limits with the experimental data published in literature[6,7,8].

It may be noted that although not easily grows length dimension, just by entering a baffle plane the transmission loss increases slightly. The experience of the author, most relevant characteristic of a muffler is the ratio between total length and its diameter dimension.

Acknowledgement

The authors acknowledge the support of the Managing Authority for Sectorial Operational Programme for Human Resources Development (AMPOSDRU) for creating the possibility to perform these researches by Grant POSDRU/89/1.5/S/62557.

References

[1] A.F. Seybert, C.Y.R. Cheng *Application of the Boundary Element Method to Acoustic Cavity Response and Muffler Analysis*, Journal of Vibration, Acoustics, Stress and Reliability in Design, January 1987, Vol. 109/15

[2] O. Vasile, *Reactive Silencer Modeling with Boundary Element Method and Experimental Study*, Proceedings of the 5th International Vilnius Conference, "Knowledge-Based Technologies and OR Methodologies for Strategic Decisions of Sustainable Development" (KORDS 2009), September 30-October 3, 2009, Vilnius, Lithuania, pp. 544-549, ISBN 978-9955-28-482-6.

[3] O. Vasile, *Contributions on modelling of noise and vibration control*, PhD thesis, 2009.

[4] D.D. Davis Jr., G.M. Stokes, D. Moore, G.L.Stevens Jr., *Theoretical and experimental investigation of mufflers with comments on engine-exhaust muffler design*, NACA Report 1192, 1954.

[5] *VNoise. Theoretical Manual. An Introduction to Boundary Elements Methods for Acoustics, September, 2003*

[6] Munjal, M.L., *Acoustics of Ducts and Mufflers With Application to Exhaust and Ventilation System Design*, John Wiley & Sons, Inc., 1987, ISBN 0-471-84738-0.

[7] Chung, J.Y. and Blaser, D.A., 1980, *Transfer Function Method of Measuring In-Duct Acoustic Properties: I. Theory and II. Experiment*, Journal of the Acoustical Society of America, Vol. 68, No. 3, pp. 907-921.

[8] Davies, P.O.A.L., 1993, *Realistic Models for Predicting Sound Propagation in Flow Duct Systems*, Noise Control Engineering Journal, Vol. 40, pp. 135-141.

Address:

- Assist.Prof. Dr. Eng. Ovidiu Vasile, "Politehnica" University of Bucharest, Department of Mechanics, Splaiul Independentei, nr. 313, 060042, Bucharest, Romania vasile@cat.mec.pub.ro