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Comparative Analysis of Visco-elastic Models with Variable Parameters

The paper presents a theoretical comparative study for computational behaviour analysis of vibration isolation elements based on viscous and elastic models with variable parameters. The changing of elastic and viscous parameters can be produced by natural timed evolution demotion or by heating developed into the elements during their working cycle. It was supposed both linear and non-linear numerical viscous and elastic models, and their combinations. The results show the importance of numerical model tuning with the real behaviour, as such the characteristics linearity, and the essential parameters for damping and rigidity. Multiple comparisons between linear and non-linear simulation cases dignify the basis of numerical model optimization regarding mathematical complexity vs. results reliability.

Keywords: *Visco-elastic model, variable parameters, spectral characteristic*

1. Introduction

The actual researches on isolation and protection against the undesirable effects of the vibration are attempted on a few main directions, but a common way to simulate and analyze the behaviour supposes a single degree of freedom model. Hereby, one of the most important components of such model is the visco-elastic characteristics approach of isolation devices.

The viscous component provides the energy losses into the isolation system during the working time. Including the thermal effects through the mechanic - thermal transition results a permanent changes of isolation characteristics, because of the rigidity dependences with the instantaneous temperature.

The Voight visco-elastic linear/non-linear model is the basic component of almost simulations. But, supposing the thermal shifting of the characteristic, the suitability of this model become undesirable at times [1].

In this study the authors present a comparative analysis of spectral characteristics for a set of well known four rheological models used in dynamic numerical simulations, with dignify on the best type for usual mechanical applications depending by their dynamic range.

2. Theory Basics

The simplified model with single degree of freedom was considered for this approach of dynamic system behaviour. A schematic diagram for practical application of vibration isolation is depicted in **Figure 1**. This simplified computational model for simulation and analysis of dynamic behavior contain a single mass noted m_t but it is the sum of all masses included in the analyzed technological equipment. This hypothesis is valid also for the rigidity k_t^{eq} [2].

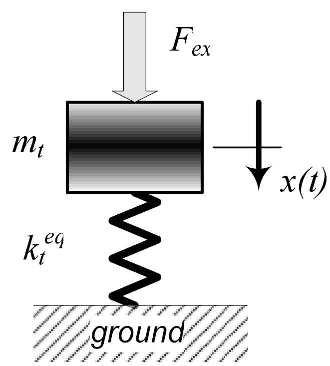

Figure 1. Schematic diagram of computational SDOF model

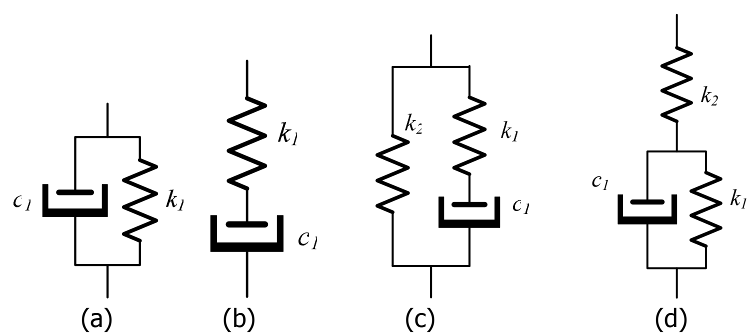

Figure 2. The basic rheological models

Note that it was considered the complex modulus of isolation system. Hereby both the recoverable (elastic) and non-recoverable (viscous) components was simulated. On the diagrams in **Figure 1** this fact was denoted through complex rigidity k_t^{eq}. The common rheological models used for simulations are: Voight model (see **Figure 2a**), Maxwell model (see **Figure 2b**), Zener model (see **Figure 2c**) and Kelvin model (see **Figure 2d**). In this paper was briefly presented the main results of a comparative study between these models, regarding the complex modulus evolution with dynamic range of excitation. Hereby it was dignified both storage modulus, and loss modulus. Evaluation of these parameters was performed using the characteristic equation for each type of rheological models. Hereby, the Voight model equation is

$$k_t^{eq} = k_1 \left(1 + i\omega\tau_{11}\right), \tag{1}$$

the Maxwell model equation is

$$k_t^{eq} = k_1 \frac{i\omega\tau_{11}}{1 + i\omega\tau_{11}}, \tag{2}$$

the Kelvin model equation is

$$k_t^{eq} = k_2 \frac{\left(\tau_{12}/\tau_{11}\right) + i\omega\tau_{12}}{1 + \left(\tau_{12}/\tau_{11}\right) + i\omega\tau_{12}}, \tag{3}$$

and the Zener model equation is

$$k_t^{eq} = k_2 \frac{1 + i\omega\left(\tau_{11} + \tau_{12}\right)}{1 + i\omega\tau_{11}}. \tag{4}$$

The parameters $\tau_{11} = c_1/k_1$ and $\tau_{12} = c_1/k_2$ on characteristic equations denotes direct and cross time constants of visco-elastic models. It has to be mentioned that the real rigidity is the main parameter for each model, hereby it was taken the unique elastic component for Voight and Maxwell models, and the additional elastic component for Zener and Kelvin models (supposing this supplementary component add the major behavior changes comparative with the basic models).

The characteristic equations of these four models in respect with force - displacement was also considered. Hereby, for each type of model, these are

Maxwell: $$F + \tau_{11}\dot{F} = c\dot{x} \tag{5}$$

Voight: $$F\frac{1}{k_1} = x + \tau_{11}\dot{x} \tag{6}$$

Zener: $$F + \tau_{11}\dot{F} = k_2 x + \tau_{11}\left(k_1 + k_2\right)\dot{x} \tag{7}$$

Kelvin: $$F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) + \tau_{11}\frac{1}{k_2}\dot{F} = x + \tau_{11}\dot{x} \tag{8}$$

where F and x means external force and displacement applied, respectively acquired by each model (with their derivatives with respect in time).

3. Comparative Computational Analysis

The main results of this analysis were conducted on two directions as follows: unitary ratio of rigidities with changes of damping, respectively unitary value of damping with rigidities ratio changes. Diagrams of modulus and loss angle was depicted in **Figure 3** (for the first analysis case; left side acquire *10^2*, middle *1* and right side *10^{-2}* values for damping), respectively in **Figure 4** (for the second case; left side acquire *10^{-3}*, middle *1* and right side *10^3* values for rigidities ratio). For all the simulations $k_1=10^3$.

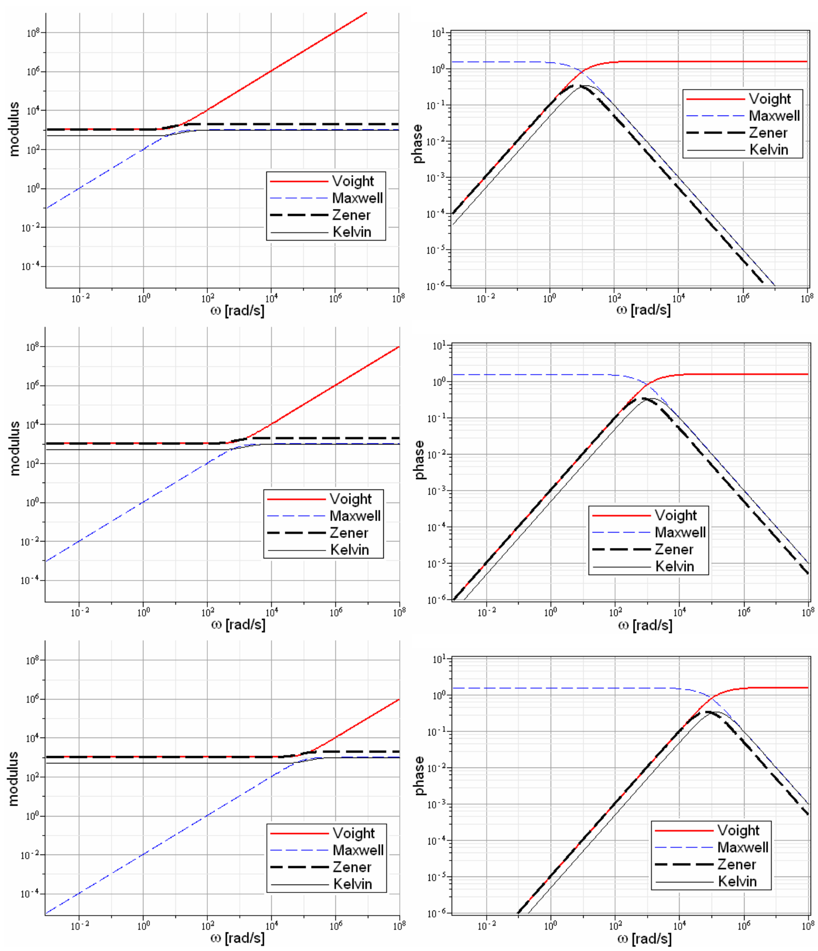

Figure 3. Modulus and loss angle evolution for damping modification

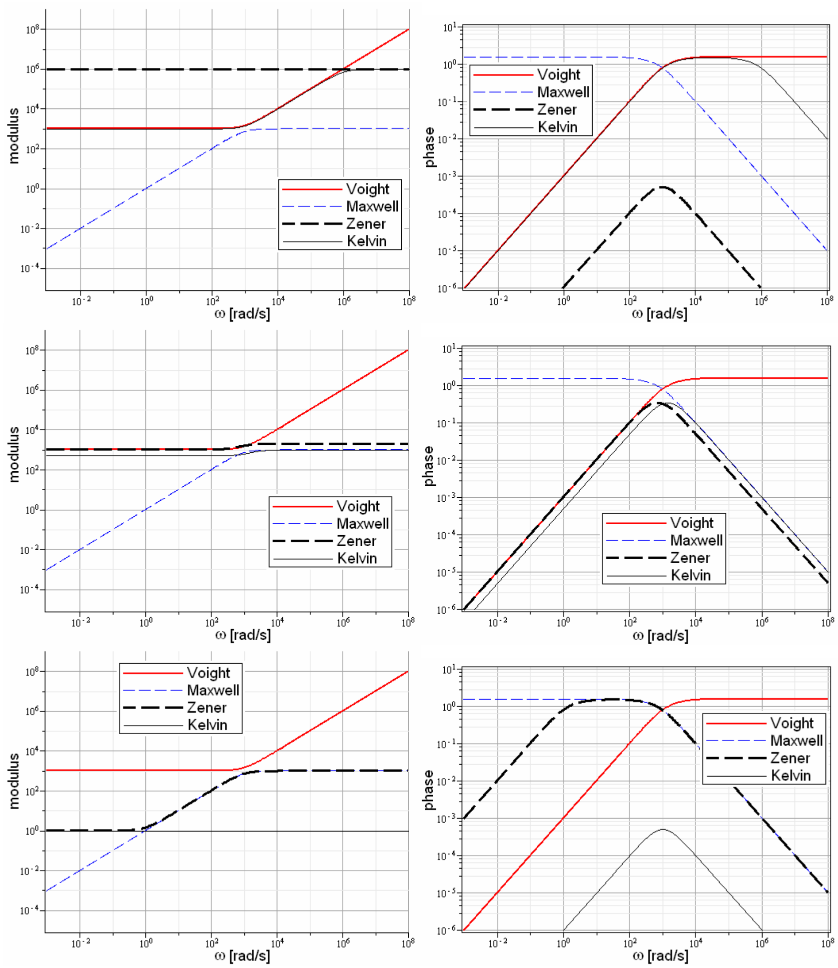

Figure 4. Modulus and loss angle evolution for rigidity modification

Analysis of these diagrams dignifies the shifting of working characteristics with changes of real values of rigidities and damping. This fact leads to a wrong evolution of some models, taking into account the frequency of external dynamic signal and the cut-off frequency of models. Choosing of a proper model for a practical analysis case have to consider the linear zone of characteristic, regarding the frequency interval of the external action. Simple visco-elastic models have a linear characteristic up to or beyond the cut-off frequency. Also, from the presented diagrams results that even the complex rheological models provide a linear characteristic for a large frequency interval, it have a changing of modulus value on the area around the cut-off frequency.

4. Concluding Remarks

The result of this analysis indicates clearly that neither model from the considered types can provide a full linear characteristic for a large interval of excitation frequency. Taking into account the fact that whatever of technical systems working with a relative restricted dynamics evolution, at least form the viewpoint of their frequency parameter, it is possible to adopt one proper model that can provide a linear characteristics on a desirable area. If two or more models accomplish this requirement, it must be preferred the complex one because of its accuracy in simulation, or the simple one whether the computations in ensemble need to be reduced.

Acknowledgments

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