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The Ultrasound Propagation Analysis through Two Mediums in Case of High Energy Piezoelectric Transducer

The high energy ultrasound it is generated by piezoelectric elements which are excited from electric signal obtained from an electronic generator. The transmission of this high energy it is obtained by transmission element or concentrator element in function of what we want to do with this energy especial utilized in no conventional technology. The efficiency work is very important aspect because the energy utilizes has high values. In this paper it is presented methods to calculate the elements components necessary to generates and transmits the high ultrasound energy: piezoelectric transducers and transmission / concentrator elements. It is presented the experimental results obtained with the theory presented. Original contribution consists by method used and suggestive graphics for appreciation of variations parameters.

***Keywords**: ultrasound, vibration, propagation, piezoelectric-transducer*

1. Introduction

In ultrasound waves practice [1] we have two important cases.

The case of the transmission/concentrator elements [2], when the ultrasound waves pass through one medium (usual alloy aluminum or titan) and the case of ultrasound piezoelectric transducer [3], when the ultrasound waves pass through two mediums (usual alloy aluminum - piezoelectric element).

2. The principle of operation

In practice [4], we have two propagation cases of high energy ultrasound waves: the case of one propagation medium and the case of two propagation mediums.

2.1 The case of one medium propagation

In this case (transmission/amplification) we have two situations.

a) The case when section element is constant (transmission element).

For to transmit ultrasound energy it is necessary that the long of metallic element L to be a multiple of $\lambda/2$: $L=n\cdot\lambda/2$; $\lambda=v/f$ where v/λ is the propagation speed/the waves long of ultrasound waves through material; f is the work frequency of ultrasound field and n is a number.

The speed v depends by parameters:

ρ- the material density;

Y- the elasticity module;

σ-the Poisson coefficient.

If the elastic longitudinal waves are propagating into non limit medium, the speed v, is given by relation:

$$v=\sqrt{\frac{Y\cdot(1-\sigma)}{\rho\cdot(1-\sigma)\cdot(1-2\sigma)}}\,. \tag{1}$$

In the case of a bar, which have the same diameter, this expression it is simplified, and it is given by expression: $v=\sqrt{\frac{Y}{\rho}}$

b) The case when we have a variable section of diameter (concentrator and transformation elements).

It start [5] from propagation relation of longitudinal plane waves through bars having variable section, given by relation:

$$\frac{d}{dx}\left[Y\cdot A(x)\cdot\frac{d\xi}{dx}\right]=\rho\cdot A(x)\cdot\frac{d^2\xi}{dt^2}\,, \tag{2}$$

where: $\xi(x,t)$-represent the vibration displacement of material points;

A(x)-represent the section transversal area;

Y- represent the elasticity module (Young module);

ρ- represent the density of material.

Considering a single mode vibration of frequency (usually longitudinal vibration), the elastic deformation of propagation wave, decrees by an exponential law, given by relation: $\xi(x,t)=y(x)e^{j\omega t}$, where $y(x)=Y_0\cdot\sin 2\pi x/\lambda$ represent the vibration amplitude along of bar, noted by x.

In case when Y=ct. (we have the same material, which have the same elasticity module) we put the $\xi(x,t)$ value in propagation formulas (2) and we obtain [6]:

$$Y\cdot e^{j\omega t}\cdot\frac{d}{dx}\left[A(x)\cdot\frac{dy(x)}{dx}\right]=-\rho\cdot A(x)\cdot\omega^2\cdot e^{j\omega t}\cdot y(x)$$

$$\Rightarrow \frac{d^2 y(x)}{dx^2} + \frac{1}{A(x)} \frac{dA(x)}{dx} \frac{dy(x)}{dx} + \frac{\rho \cdot \omega^2}{Y} y(x) = 0 . \tag{3}$$

We have: v= $\sqrt{\frac{Y}{\rho}}$ (the bar, have the same diameter).

Result:

$$\Rightarrow \frac{d^2 y(x)}{dx^2} + \frac{d[\ln A(x)]}{dx} \cdot \frac{dy(x)}{dx} + \frac{\omega^2}{v^2} \cdot y(x) = 0 , \tag{4}$$

2.2 The case of two propagation mediums.

It is the case of a piezoelectric transducer [7], formed by two piezoelectric pastilles which are catch between two metallic blocks (named director and reflector) with help, in general, of a central screw.

The reflector is made from steel, aluminum, bronze, naval brass (having a high density of material - ρ). The director is made from titanium, aluminum or magnesium composite (having a low density of material -ρ).

These choice of materials help us to obtain a directivity of energy propagation from reflector to director.

The long of hole transducer must be λ/2 - figure1:

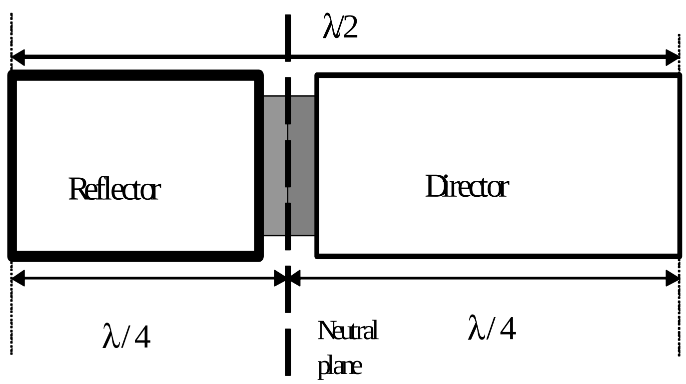

Figure 1. The piezoelectric transducer.

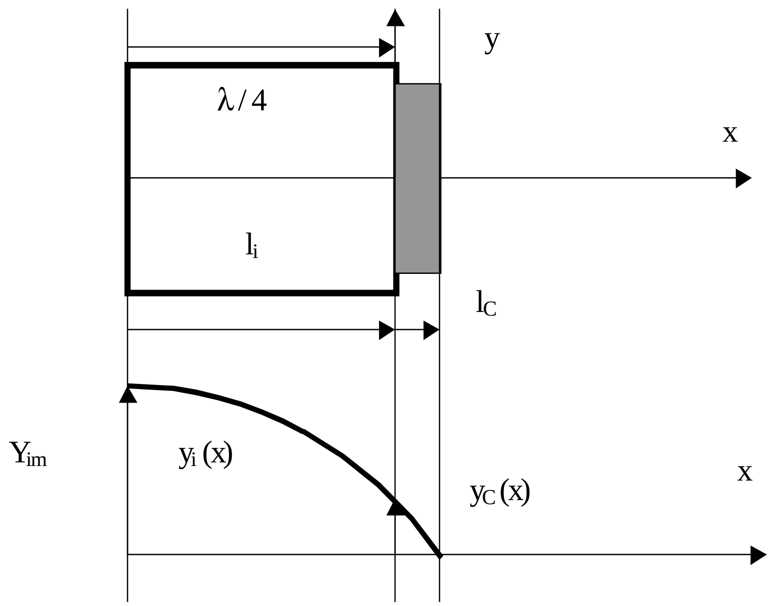

Figure 2. The partial transducer - reflector.

It is considerer a partial transducer-reflector having the shape, given in figure2.

It is note:

$$K_i = \frac{2\pi}{\lambda_i} = \frac{2 \cdot \pi \cdot f}{v_i} = \frac{\omega}{v_i}$$
$$K_C = \frac{2\pi}{\lambda_C} = \frac{2 \cdot \pi \cdot f}{v_C} = \frac{\omega}{v_C}, \quad (5)$$

where K_i– referrer to the end piece and K_C– referrer to the ceramic material.

The displacement variation y is given by relations:

$y_i(x)=y_{iM}\cdot\cos[K_i(x+l_i)]$ - for the piece end, reflector,

(6)

$y_C(x)=y_{CM}\cdot\sin[K_C(x-l_C)]$- for the ceramic material,

At limit separation of end piece-ceramic the curve must be continue, so must have:

a) the displacements y(x) for x = 0 must be the same, so: $y_i (0) = y_C (0)$

b) the force which action the ceramic over end piece $F_C(0)$ must be equal with reaction force of end piece $F_i(0)$, so: $F_C(0) = F_i(0)$

With these two limit conditions it is obtains:

a) $\rightarrow y_{iM} \cdot \cos K_i l_i = -y_{CM} \cdot \sin K_C l_C$

b) $\rightarrow$ It is note that:

- Y_C – represents the ceramic elasticity module; - Y_i – represents the end piece elasticity module; - A_C- represents the ceramic transversal area;

- A_i- represents the end piece transversal area.

We have:

$$F(x)=Y \cdot A \cdot \frac{dy(x)}{dx}, \tag{7}$$

$$\text{so: } F_i(x)=Y_i \cdot A_i \cdot \frac{dy_i(x)}{dx} \quad \text{and} \quad F_C(x)=Y_C \cdot A_C \cdot \frac{dy_C(x)}{dx}, \tag{8}$$

Putting these values in (6) relation, it obtains:

$$F_i(x) = -Y_i \cdot A_i \cdot y_{iM} \cdot K_i \cdot \sin[K_i(x+l_i)]$$

$$F_C(x) = Y_C \cdot A_C \cdot y_{CM} \cdot K_C \cdot \cos[K_C(x-l_C)], \tag{9}$$

By equalizer of these two conditions (for x=0), it obtains:

$$Y_C \cdot A_C \cdot y_{CM} \cdot K_C \cdot \cos(-K_C \cdot l_C) = -Y_i \cdot A_i \cdot y_{iM} \cdot K_i \cdot \sin K_i \cdot l_i, \tag{10}$$

From a) condition it obtains:

$$\frac{y_{iM}}{y_{CM}} = -\frac{\sin K_C \cdot l_C}{\cos K_i \cdot l_i}, \tag{11}$$

From b) condition it obtains:

$$\frac{Y_C \cdot A_C \cdot K_C}{Y_i \cdot A_i \cdot K_i} = -\frac{y_{iM}}{y_{CM}} \cdot \frac{\sin K_i \cdot l_i}{\cos K_C \cdot l_C} = -\left(-\frac{\sin K_C \cdot l_C}{\cos K_i \cdot l_i}\right) \cdot \frac{\sin K_i \cdot l_i}{\cos K_C \cdot l_C} =$$

$$= \frac{\sin K_C \cdot l_C}{\cos K_C \cdot l_C} \cdot \frac{\sin K_i \cdot l_i}{\cos K_i \cdot l_i} = \operatorname{tg} K_C l_C \cdot \operatorname{tg} K_i l_i, \tag{12}$$

We have the relations:

$v=\sqrt{\frac{Y}{\rho}}$ - bars; $K_i = \frac{\omega}{v_i}$; $K_C = \frac{\omega}{v_C}$ - by definition

So we can write:

$$Y_C \cdot K_C = v_C^2 \cdot \rho_C \cdot \frac{\omega}{v_C} = \omega \cdot \rho_C \cdot v_C,$$

$$Y_i \cdot K_i = v_i^2 \cdot \rho_i \cdot \frac{\omega}{v_i} = \omega \cdot \rho_i \cdot v_i, \tag{13}$$

The equation (12) became:

$$tgK_C l_C \cdot tgK_i l_i = \frac{\rho_C \cdot v_C \cdot A_C}{\rho_i \cdot v_i \cdot A_i}, \tag{14}$$

It know that $\omega=2\pi f$; $\rho_C v_C$; $\rho_i v_i$; A_C;A_i. So, it choice l_C or l_i and result the other. Usual it choice l_C, like a compromise between economic reasons (slim ceramic) and efficiency reasons. Result the l_i value.

So we find the long of reflector piece.

To resolves the equation it is difficult and so resolves it by graphic.

The equation (14) may be written under form:

$$tg\frac{\omega l_i}{v_i}\cdot tg\frac{\omega l_C}{v_C}=q_i=\frac{\rho_C\cdot v_C\cdot A_C}{\rho_i\cdot v_i\cdot A_i}=\frac{Z_C}{Z_i}, \tag{15}$$

because: $K_i=\omega/v_i$;$Z_C=\rho_C\,v_C$; $K_C=\omega/v_C$; $Z_i=\rho_i\,v_i$

Z_C and Z_i represents the acoustic impedance of ceramic material and reflector material

Result the curves given in fig. 3 where it is noted:

- ρ_C–the ceramic density;
- l_C-the ceramic thickness;
- v_C-the propagation speed through ceramic;
- A_C-the ceramic surface;
- ρ_i – the metallic density;
- l_i - the metallic thickness;
- v_i – the propagation speed through metal;
- A_i – the metal surface.

Multiplier the angle by 2 / π and obtain:

$$\frac{2}{\pi}\cdot\frac{\omega l_i}{v_i}=\frac{2}{\pi}\cdot\frac{2\pi f l_i}{v_i}=\frac{4l_i}{\lambda_i} \qquad \text{and}$$

$$\frac{2}{\pi}\cdot\frac{\omega l_C}{v_C}=\frac{2}{\pi}\cdot\frac{2\pi f l_C}{v_C}=\frac{4l_C}{\lambda_C}, \tag{16}$$

Because we have: $v_i=\lambda_i\cdot f$ and $v_C=\lambda_C\cdot f$.

Result:

$$tg\frac{4l_C}{\lambda_C}\cdot tg\frac{4l_i}{\lambda_i}=q_i, \tag{17}$$

In usual range, [8] $q_i\in(0{,}4\div4)$ it marking the curves $4l_i/\lambda_i$ function of de $4l_C/\lambda_C$ having like parameter $q_i=\dfrac{\rho_C\cdot v_C\cdot A_C}{\rho_i\cdot v_i\cdot A_i}$.

In function of used metal we find a q_i value. Known $4l_C/\lambda_C$, will result $4l_i/\lambda_i$.

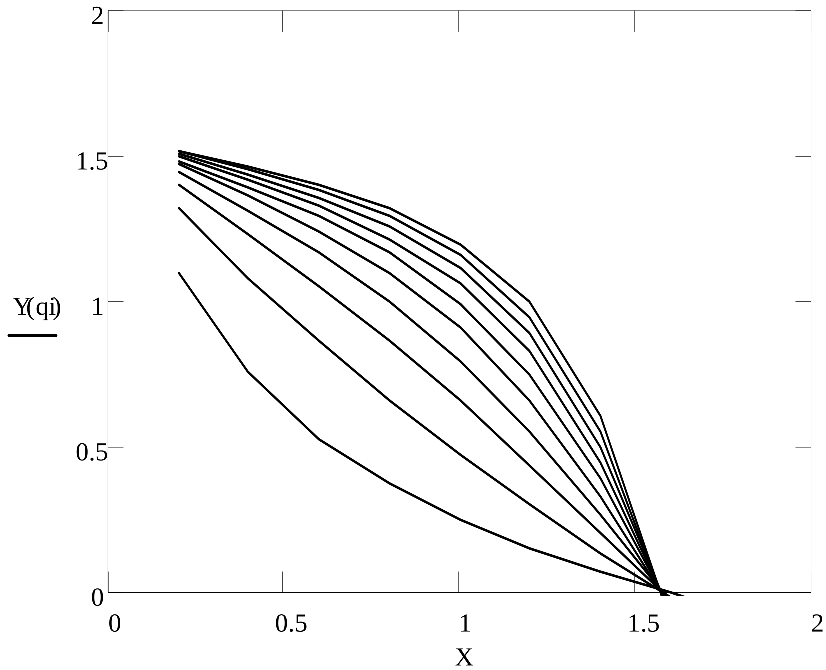

Figure 3. The $4l_i$ / λ_i curves [$Y(q_i)$] in function of $4l_C$ / λ_C (x) having q_i like parameter.

In figure 3 it is noted $X = 4l_C / \lambda_C$ and $Y=4l_i / \lambda_I$.

The value $q_i = 0.4$ it is obtained to lower of these set of curves and $q_i = 4$ it is obtained to higher of these set of curves.

For $q_i = 1$ the shape of variation is linear.

So we may find out l_i. The curves are given in fig. 3.

By utilization of end material so to have $q_i = \dfrac{\rho_C \cdot v_C \cdot A_C}{\rho_i \cdot v_i \cdot A_i} \neq 1$ it obtains an amplification/amortization of ultrasound intensity at respective end surface, with amplification coefficient G_i given by relation (18):

$$G_i = q_i^2 - \left(q_i^2 - 1\right)\sin^2 \frac{\omega_S l_C}{v_C} = q_i^2 - \left(q_i^2 - 1\right) \cdot \sin^2 \frac{4 \cdot l_c}{\lambda_c}, \tag{18}$$

where $\omega_S = 2\pi f_S$.

The amplification is higher when:

a) The $4l_C/\lambda_C$ ratio is small. From this remark, result that a higher efficiency it is obtains if l_C is small, so the ceramic thickness must be small. Usually it take

$4l_C/\lambda_C<0{,}25$ or $l_C<$ o,o6 λ_C . A too small thickness affects the coupling factor K_{eff} from which depend the transducer efficiency.

b) $q_i = \dfrac{\rho_C \cdot v_C \cdot A_C}{\rho_i \cdot v_i \cdot A_i} > 1$. From this expression result that we have amplification if we have:

- $\rho_i<\rho_C$, director density < ceramic density;
- $v_i<v_C$, director velocity < ceramic velocity;
- $A_i<A_C$, director area < ceramic area .

From constructive reasons [9], (for don't broke ceramics) the last condition don't be realized.

From density condition it choose aluminum alloy for to obtain an amplification of vibrations.

c) the variation of effective coupling coefficient K_{eff} depends of $4l_C/\lambda_C$ variable having like parameter q_i from graphic given in fig.5a.

It's observes that K_{eff} subtract when ceramic thickness l_C subtract.

For a piezoelectric transducer we have:

- the reflector which has $q_i<1$, because $\rho_i>\rho_C$ (ρ_i for steel, aluminum alloy, bronze). So we have an increasing of K_{eff}
- the director which has $q_i>1$, because $\rho_i<\rho_c$ (ρ_i for aluminum alloy, titan, magnesium alloys). So we have a subtracting of K_{eff}.

For hole ensemble of piezoelectric transducer will have qi = 1.

It's observe that if $4l_C/\lambda_C<0{,}15$, than $K_{eff}^2<0{,}25 \cdot K_{33}^2$, where K_{33} is ceramic coupling coefficient for vibration mode utilizes. Chose $K_{eff}^2 \geq 0{,}25 \cdot K_{33}^2$

result: $\dfrac{4 \cdot l_C}{\lambda_C}>0{,}15$ or $l_C>\dfrac{\lambda_C}{26{,}6}=0{,}03\lambda_C$.

Taking for ceramic the value v=3100m/s will obtain, for a frequency f=20kHz: $\lambda_C=\dfrac{v}{f}=\dfrac{3100}{20 \cdot 10^3}=155mm$, so to obtain a coupling factor $\left(\dfrac{K_{eff}}{K_{33}}\right)^2 \geq 0{,}25$, the ceramic thickness may accomplish the condition: $l_C \geq \dfrac{155}{26{,}6}=5{,}8mm$

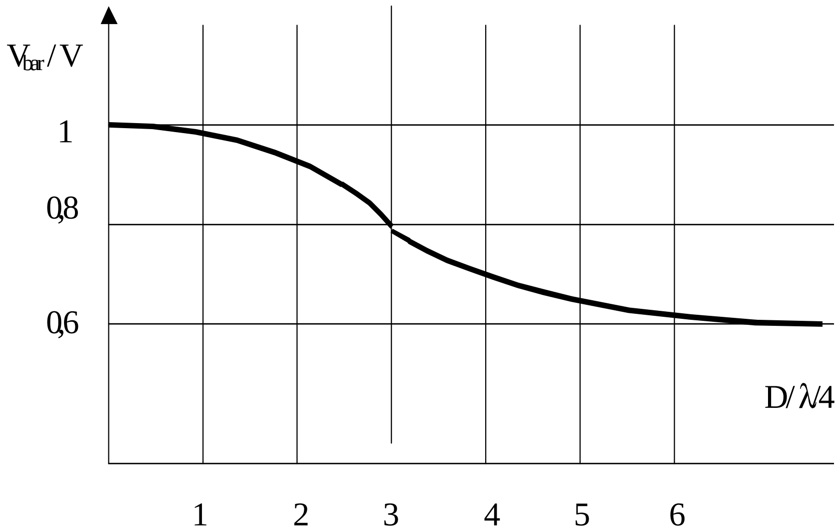

Figure 4. The propagation speed variation through bars having miscellaneous D sections.

In fig. 5 b) it is presented the relation between ξ' and ξ'' ($\xi = \xi' - j\xi''$, where ξ represents radiation coefficient) and diameters D_1 and D_2 of front radiation 1 and 2.

The labeled area show the range where the real part of radiation coefficient ξ' becomes unacceptable very low.

The ultrasonic intensity through liquid, near radiation surface, it is given (only plane wave through liquid) by relation:

$$I_{amax} = \frac{1}{2} \cdot (\omega_S \cdot Y_1)^2_{amx} \cdot \rho_{ma} \cdot v_a ; \qquad (19)$$

where Y_1 represents the vibration amplitude, and

$$I_{ai\,max} = \frac{1}{2} \cdot G_i \cdot \left(\frac{T_{C\max}}{\rho_{mC} \cdot v_C} \right)^2 \cdot (\rho_{ma} \cdot v_a)_i , \qquad (20)$$

At lower frequency we haven't only plane waves.

This effect must be take in account by the right member of these two equations which must be multiply by real part of radiation coefficient ξ'.

The ξ factor subtracts more than linear at low frequency. This represents a limitation.

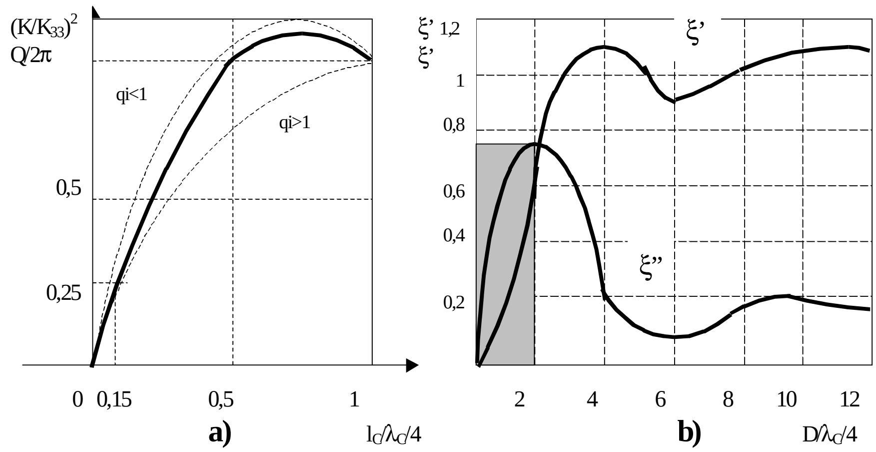

Figure 5. The coupling variation coefficient in function of l_C and the radiation in function of diameter D.

For a minimum admissible by approximate 0.75 (showed in fig. 5.b) the minim diameter D of irradiative face must be $\lambda/2$, which correspond with work liquid (in general water).

A considerable mass it is attached at transducer by liquid charge so the transducer is considerable unbalanced (ξ'' curve).

3. Experimental results

In case of piezoelectric transducers an important role in efficiency work it is represented by acoustic charge.

This acoustic charge can move the neutral plane from insides of piezoelectric ceramics. When this action is very strong the piezoelectric transducer must be equipped with speed acoustic transformation.

For acoustic charge measurement of transducer one uses a K7103 Velleman digital oscilloscope coupled with PC computer.

With a K8016 Velleman digital generator, one generates a sinusoidal signal with variable frequencies from 100 Hz by step. It was followed the response of transducer which was charged with different acoustic chargers.

On digital generator output it was connected a resistance equal with 1,8kΩ in series with transducer and output amplitude of digital generator was $5V_{vv}$.

The acoustic charge – air or an aluminum/steel cylinder – was laid on surface of the transducer.

It can seen that when the acoustic charge is air, the minimum of frequency characteristic transfer is for 25,5 kHz – r = 31 Ω –, for aluminum 21,4kHz – r = 396 Ω – and for steel 21,2 kHz – r = 960 Ω.

It was find the following numerical values for these 3 cases (frequency / peak tension / attenuation):

Table 1.

Air			Aluminum			Steel		
25000	0.544	-5.289	20900	1.173	1.385	20700	1.399	2.917
25100	0.455	-6.833	21000	1.105	0.867	20800	1.389	2.854
25200	0.353	-9.047	21100	0.998	-0.013	20900	1.362	2.681
25300	0.234	-12.602	21200	0.832	-1.601	21000	1.323	2.432
25400	0.155	-16.212	21300	0.579	-4.745	21100	1.220	1.727
25500	0.044	-27.055	21400	0.444	-7.051	21200	0.872	-1.188
25600	0.220	-13.163	21500	0.928	-0.647	21300	1.351	2.611
25700	0.418	-7.580	21600	1.361	2.676	21400	1.606	4.117
25800	0.632	-3.986	21700	1.534	3.718	21500	1.563	3.878
25900	0.858	-1.332	21800	1.573	3.936	21600	1.533	3.711
26000	1.074	0.623	21900	1.579	3.969	21700	1.501	3.527

4. Conclusion

The paper presents a practical guide to design the piezoelectric transducer.

In this case the neutral plane of vibration pass through ceramics pieces.

This is for a maximum transformation efficiency of electric signal into mechanical vibration.

The piezoelectric ceramics must be emplaced in nodal plane where the displacements are null y=0 and mechanical tensions are maximum T=max.

Acknowledgments

With calculus presented in this paper it is possible:

- To calculate the dimensions of piezoelectric transducer functions by: materials used, frequency work.
- To appreciate the influence of different parameters to final response;
- To calculate of transformation elements used to transmit a high energy to work medium

- To optimize hole chain transformations beginning from electric-mechanical transformation and finishing with unconventional technology which use ultrasound energy.

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