ANALELE UNIVERSITĂŢII
"EFTIMIE MURGU" REŞIŢA
ANUL XVII, NR. 1, 2010, ISSN 1453 - 7397

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Optimisation Methods for Cam Mechanisms

Abstract. *In this paper we present the criteria which represent the base of optimizing the cam mechanisms and also we perform the calculations for several types of mechanisms. We study the influence of the constructive parameters in case of the simple machines with rotation cam and follower (flat or curve) of translation on the curvature radius and that of the transmission angle. As it follows, we present the optimization calculations of the cam and flat rotation follower mechanisms, as well as the calculations for optimizing the cam mechanisms by circular groove followers' help. For an easier interpretation of the results, we have visualized the obtained cam in AutoCAD according to the script files generated by a calculation program.*

Keywords*: cam, curvature radius, constructive parameters, pressure angle, circular grove.*

1. Optimisation criteria

The term of optimization comes from the Latin word – optimus, which is the superlative of good, which means the best, very good, properly indicated, suited etc. According to Explaining Dictionary of Romanian Language, by optimization is understood – technically – the ensemble of scientific research (papers) that is looking for the best option in finding a solution for a problem or, according to another definition, the process of constant improving until the best solution it is reached.

Mathematically, by *optimization* is understood the reasoning, the calculus permits in finding values of one or more parameters which correspond to the maximum of a function.

In the case of a cam mechanism, one of the optimization criteria is *the criteria of the curvature radius*, according to that, the curvature radius of the cam in the case of a follower which is flat or has a positive curvature radius must be positive and even higher then an imposed value.

Another criteria that must be taken into account is that of the *pressure angle* (noted with α) which must fulfill the condition $\alpha < \alpha_{cr}$, where $\alpha_{cr} = 30^0$ at increasing and 60^0 at decreasing.

On the basis of this two criteria will be obtained better condition for the complex cam mechanisms to function.

Next we will define as a *technically functional cam* the cam that is obtained by classic procedures of processing of a tool machine, having the curvature radius positive and that respects the condition of the critical pressure angle ($\alpha < \alpha_{cr}$).

2. The optimisation of simple cam mechanisms with translation follower

There is considered the case of a flat follower (fig. 1), case where the parametrical coordinates of the cam are:

$$\begin{aligned} x &= (R_0 + s)\sin\varphi + s'\cos\varphi; \\ y &= (R_0 + s)\cos\varphi - s'\sin\varphi, \end{aligned} \tag{1}$$

and

$$O_2A = s'. \tag{2}$$

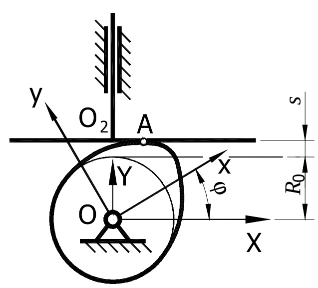

Figure 1. Mechanism with rotation cam and flat translation follower.

From (1) are deducted:

$$\begin{aligned} x' &= (R_0 + s + s'')\cos\varphi; \\ y' &= -(R_0 + s + s'')\sin\varphi; \end{aligned} \tag{3}$$

$$\begin{aligned} x'' &= (s' + s'')\cos\varphi - (R_0 + s + s'')\sin\varphi; \\ y'' &= -(s' + s'')\sin\varphi - (R_0 + s + s'')\cos\varphi, \end{aligned} \tag{4}$$

and it’s also deducted the expression of the curvature radius:

$$R_c = \frac{[(x')^2 + (y')^2]^{3/2}}{|x'y'' - x''y'|}; \quad R_c = R_0 + s + s''. \tag{5}$$

If, for example, the displacement law is:

$$s = h_0(1 - \cos 2\varphi). \tag{6}$$

Then the curvature radius is

$$R_c = R_0 + h_0 + 3h_0 \cos 2\varphi \tag{7}$$

and becomes minimum for $\varphi = \frac{\pi}{2}$ when is given by:

$$R_{c\min} = R_0 - 2h_0. \tag{8}$$

In this case, for $R_0 = 2h$ the minimum curvature radius becomes null and the cam has the shape from figure 2, *a,* and if $R_0 < 2h_0$ the curvature radius is canceled in the points where $\varphi < \frac{\pi}{2}$; $\frac{\pi}{2} < \varphi < \pi$ (fig. 2, *b*) and have negative value for $\varphi < \frac{\pi}{2}$ and the cam becomes nonfunctional.

If we use flat follower in these conditions we can obtain cams technically functional.

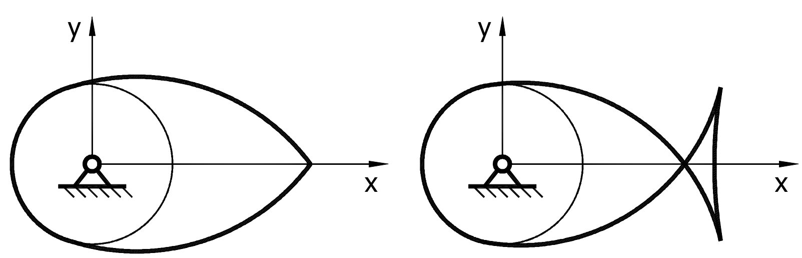

Figure 2. Nonfunctional cams with null or negative curvature radius.

Let us consider the general case where the curvature radius at the top in the case of a flat follower becomes negative $(R_0 + s_{\max} + s'') < 0$ and to determine the radius r of the circular follower so that the cam will be functionally.

The equations of the followers' groove (fig. 3) are

$$x_2 = r \sin\lambda; y_2 = -r\cos\lambda, \tag{8}$$

and from the equalities:

$$x_A = x\cos\varphi - y\sin\varphi = r\sin\lambda;$$
$$y_A = x\sin\varphi - y\cos\varphi = R_0 + r + s - r\cos\lambda, \quad (9)$$

are deducted the equations of the curvature family

$$x = (R_0 + r + s)\sin\varphi - r\sin(\varphi+\lambda) = f_1(\varphi,\lambda);$$
$$y = (R + r + s)\cos\varphi - r\cos(\varphi+\lambda) = f_2(\varphi,\lambda), ; \quad (10)$$

that depends on the parameters φ, λ.

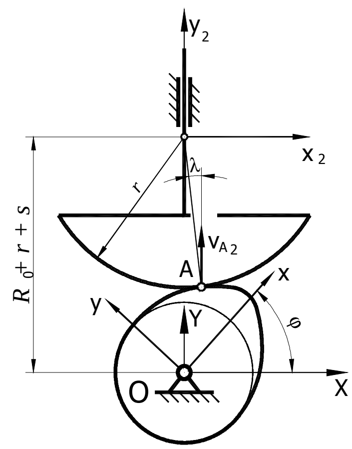

Figure 3. Mechanism with rotation cam and flat rotation follower.

The equation of the envelope checks the equation:

$$\begin{vmatrix} \frac{\partial f_1}{\partial \varphi} & \frac{\partial f_1}{\partial \lambda} \\ \frac{\partial f_2}{\partial \varphi} & \frac{\partial f_2}{\partial \lambda} \end{vmatrix} = 0, \quad (11)$$

that becomes

$$\text{tg}\,\lambda = \frac{s'}{R_0 + r + s}. \quad (12)$$

The cam equations are given by the relations (10) where λ checks the equation (12). In the top ($\varphi = \frac{\pi}{2}$) are fulfilled the conditions

$$s = s_{\max};\ s' = 0;\ \lambda = 0;\ s'' < 0;$$

$$\lambda' = \frac{s''}{R_0 + r + s_{max}};\ \lambda'' = 0; \tag{13}$$

$$x' = 0;\ y' = -(R_0 + s_{max} + \lambda' r);\ x'' = -(1-\lambda')(R_0 + s_{max} + \lambda' r)$$

and the curvature radius is

$$R_c = \frac{[(x')^2 + (y')^2]^{3/2}}{|x'' y' - x' y''|} = \frac{|y'^3|}{|x'' y'|} = \frac{(y')^2}{|x''|} = \frac{R + s_{max} + \lambda' r}{|1-\lambda'|}. \tag{14}$$

Knowing that

$$\lambda' = \frac{s''}{R_0 + r + s_{max}} < 0, \tag{15}$$

is deducted

$$R_c = \frac{(R_0 + s_{max})^2 + r(R_0 + s_{max} + s'')}{(R_0 + r + s_{max})(1-\lambda')}. \tag{16}$$

And next, by knowing that

$$1 - \lambda' < 0;\ R + s_{max} + s'' < 0, \tag{17}$$

from (16) and giving the condition $R_c > 0$ is obtained:

$$r < -\frac{(R_0 + s_{max})^2}{R_0 + s_{max} + s''}. \tag{18}$$

3. The optimisation of mechanisms with rotation follower

For the starters, is considered the mechanism with a flat follower from figure 3 that has the lengths d, R_0, b and the displacement rule:

$$\varphi_2 = \varphi_2(\varphi). \tag{19}$$

From the equations:

$$\begin{aligned} x &= x\cos\varphi - y\sin\varphi = -d + \lambda\cos\varphi_2; \\ y &= x\sin\varphi + y\cos\varphi = R_0 + b + \lambda\sin\varphi_2, \end{aligned} \tag{20}$$

are obtained the equations of the cam:

$$\begin{aligned} x &= -d\cos\varphi + (R_0 + b)\sin\varphi + \lambda\cos(\varphi - \varphi_2) - b\sin(\varphi - \varphi_2); \\ y &= d\sin\varphi + (R_0 + b)\cos\varphi - \lambda\sin(\varphi - \varphi_2) - b\cos(\varphi - \varphi_2), \end{aligned} \tag{21}$$

where

$$\lambda = \frac{d\cos\varphi_2 - (R_0 + b)\sin\varphi_2}{1 - \varphi_2'}. \tag{22}$$

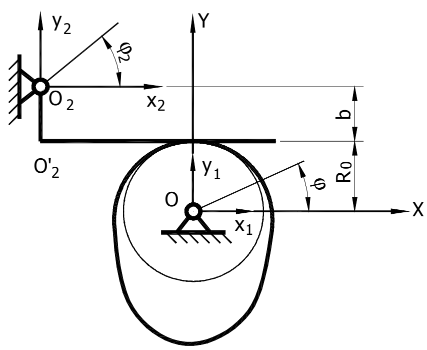

Figure 4. Mechanism with rotation cam and flat rotation follower.

The λ parameter represents the distance from the tangent point until the point O_2' and as a result the pressure angle is given by the relation:

$$\alpha = \mathrm{arctg}\,\frac{b}{\lambda}. \tag{23}$$

The curvature radius

$$R_c = \frac{[(x')^2 + (y')^2]^{\frac{3}{2}}}{|x''y' - x'y''|} \tag{24}$$

and the pressure angle α depends of the amplitude φ_2^0 of angle φ_2 as from the dimensions $R_0 + b$ and d.

The numerical analyze is made tabular with the step $\Delta\varphi = 1^0$ calculation the derivates by limited differences with sets of values for $R + b$ and *d*.

Is considered the displacement rule:

$$\varphi_2 = \frac{\pi}{20}(1 - \cos 2\varphi);\ \varphi_2' = \frac{\pi}{10}\sin 2\varphi. \tag{25}$$

On the basis of the relations (19) ÷ (24) is made a calculation program in Pascal. This works for different sets of values.

To begin interpreting much easier the obtained results it is useful in visualizing the cam that is obtained with the varying parameters.

This visualization is made in AutoCAD, based on a script file generated by the calculation program.

In figure 5 is represented the cam obtained in the case $R_0 = 10$, $b = 5$.

It is observed that in the cases where $d = 60$ and $d = 40$ that the obtained cams are nonfunctional and have negative curvature radius on some parts.

In the case where $d = 20$ the cam is technically nonfunctional although it has a continuous aspect, at $\varphi = 60°$ and $\varphi = 240°$ are noticed holes, the flat follower cannot continuing the external contour of the cams groove.

From the point of view of the pressure angle it is ok, at the lifting maneuver $\alpha_{\max} = 21.746°$ for $\varphi = 120°$.

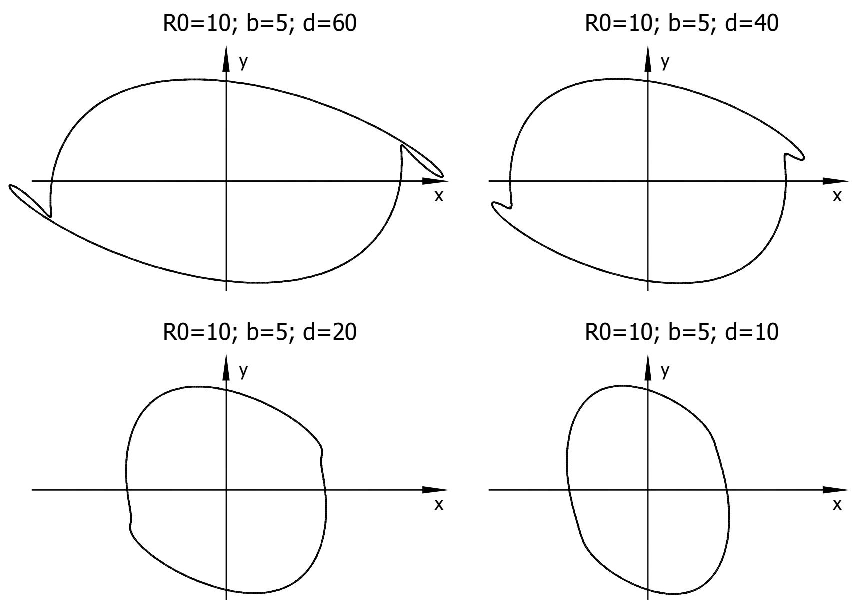

Figure 5. The cams obtained in the case b = 5.

In the case where $R_0 = 10$, $b = 5$ and $d = 20$ though it was obtained a cam technically nonfunctional, but by keeping the same dimensions can be obtained a solution technically functional by using the follower with a circular groove (fig. 6).

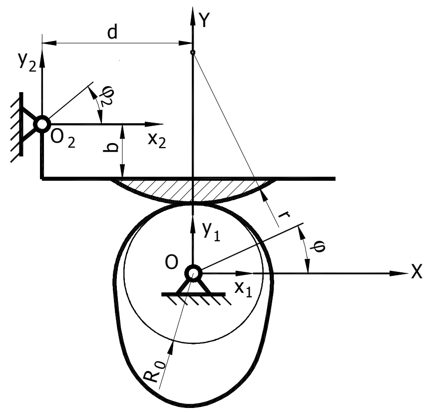

Figure 6. Mechanism with rotation cam and circularly rotation.

So are obtained:

- the equations of the follower

$$x_2 = d + r\sin\lambda;\ y_2 = y_2^o - r\cos\lambda; \tag{26}$$

- the general equations

$$\begin{aligned} x &= x\cos\varphi - y\sin\varphi = -d + x_2\cos\varphi_2 - y_2\sin\varphi_2; \\ y &= x\sin\varphi + y\cos\varphi = R_0 + r - y_2^o + x_2\sin\varphi_2 + y_2\cos\varphi_2, \end{aligned} \tag{27}$$

which are deducted the equations:

$$\begin{aligned} x &= -d\cos\varphi + (R_0 + r - y_2^o)\sin\varphi + x_2\cos(\varphi - \varphi_2) + y_2\sin(\varphi - \varphi_2); \\ y &= d\sin\varphi + (R_0 + r - y_2^0)\cos\varphi - x_2\sin(\varphi - \varphi_2) + y_2\cos(\varphi - \varphi_2); \end{aligned} \tag{28}$$

- the grabbing condition is:

$$tg\lambda = \frac{-d\cos\varphi_2 + (R + r - y_2^0)\sin\varphi_2 + d(1 - \varphi_2')}{d\sin\varphi_2 - (R + r - y_2^0)\cos\varphi_2 - y_2^0(1 - \varphi_2')}, \tag{29}$$

- the equation of the cam

$$\begin{aligned} x &= -d\cos\varphi + (R_0 + r - y_2^0)\sin\varphi + d\cos(\varphi - \varphi_2) + y_2^0\sin(\varphi - \varphi_2) - \\ &\quad - r\sin(\varphi - \varphi_2 - \lambda); \\ y &= d\sin\varphi + (R_0 + r - y_2^0)\cos\varphi - d\sin(\varphi - \varphi_2) + y_2^0\cos(\varphi - \varphi_2) - \\ &\quad - r\cos(\varphi - \varphi_2 - \lambda), \end{aligned} \tag{30}$$

where λ is deducted from the equation (29)

- the pressure angle is:

$$\alpha = -\operatorname{arctg}\frac{y_2^0 - r\cos\lambda}{d + r\sin\lambda} + \lambda . \qquad (31)$$

In the numerical case are given the values considered in the previous case of cam technically nonfunctional and there more is considered $y_2^0 \approx r - b$.

Next is represented for different cases the cam obtained in the case of a mechanism with curve follower and different curvature radius.

In figure 7 it was represented for the case $R_0 = 10$, $b = 5$, $d = 20$, with light grey the cam obtained in the case of a mechanism with flat follower and with black the cab obtained in the case of a curve follower with the curvature radius of the follower of: $r = 100,\ 50,\ 30,\ 10$.

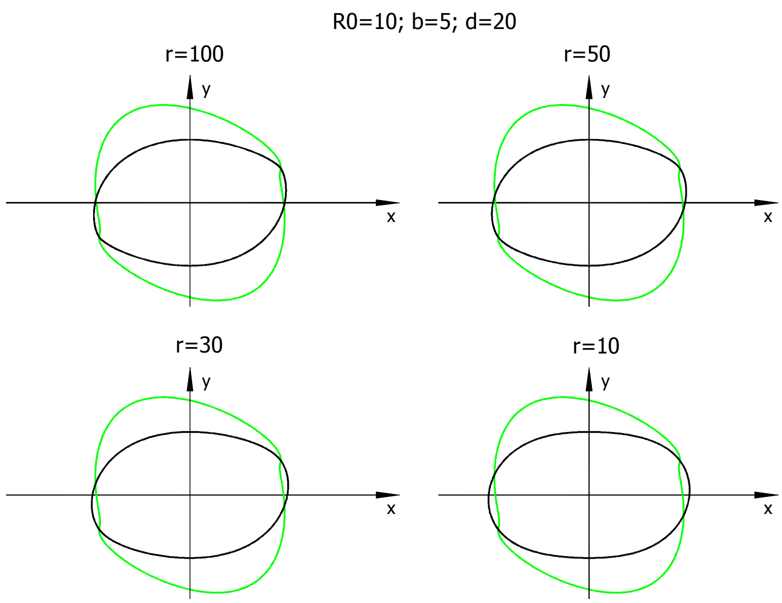

Figure 7. The cams obtained in the case b = 5 and d = 20.

In the case of the mechanism with a flat follower (the light grey cam) there is no functional cam. In all the four cases of curve follower the obtained cam is functional, the pressure angle fulfilling the condition $\alpha < \alpha_{cr}$ as both for the lifting race and for the descend race. There is observed that by lowering the curvature radius of the follower is obtained a cam with a bigger minimum curvature radius

4. Conclusions

In this paper are presented two major optimization criteria: the criteria of the minimum imposed curvature radius and the criteria of the pressure angle.

It is studied the influence of constructive parameters over the curvature radius and the pressure angle with the help of a calculation program.

For an easier interpretation of the results, the obtained cam was visualized considering the parameters that vary. This visualization is made in AutoCAD, using a script file generated by a calculation program.

In the case of optimizing the cam mechanism with a circular grooved follower, keeping the same constructive measures, were obtained technically functional cams.

It was studied the influence of a third parameter: the radius of the circular follower.

It was considered useful in overlaying the obtained cams with flat follower with the cams with a circular follower with different curvature radius of the follower.

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