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The Simulation of the Functioning of a Hybrid Vehicle

Abstract. *In the paper we present a functional model and its mathematic modeling of the movement of the mechanism that simulates the functioning of a hybrid vehicle. The mechanism used for the coupling of three electric engines of continuous current with permanent magnet stator is a planetary mechanism with a double satellite. The functional model operation is ensured by an electronic module which allows the supply voltage variation of electric motors and an electronic device. In order to study the movement of the mechanism from a mathematical point of view we have designed a mathematic model with two degrees of freedom. We obtain the solution in transition phase and in permanent phase after obtaining the differential equations of motion using Lagrange's equations.*

Keywords*: hybrid, functional mechanism, power sources, planetary mechanisms, differential equations,*

1. Solutions used in coupling mechanisms of power sources

The hybrid electric propulsion solution is a promising one for the protection of the environment by limiting the emissions.

A hybrid propulsion system is made up of a mechanically coupling system that is the link between the heat source, power supply and driving wheels of the vehicle.

The mechanical system used by the Toyota company on Prius model (fig. 1) is a planetary mechanism.

In this configuration the heat engine (MT) engages the planetary carrier arm 1, the electric car ME is acting the crown 2 and the electric generator (GE) is powered by three solar wheel.

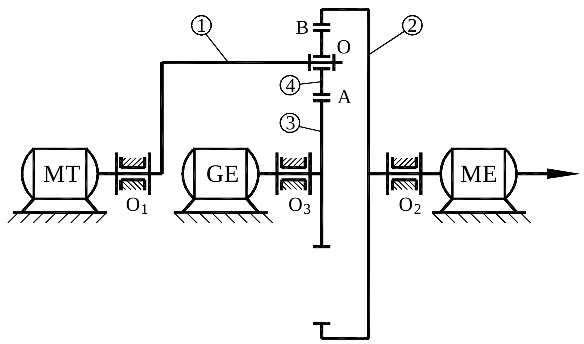

Figure 1. The kinematic scheme of planetary mechanism used by the Toyota company.

The actions of the heat engine, of the generator and those of the electric engine are:

1) The idle vehicle – the heat engine, generator and electric motor are stopped.

2) The start – the vehicle starts only by using the electric motor.

3) The start – the generator, which has thermal engine starter function, starts spinning wheel and solar thermal engine; once the heat engine has been started, the generator produces electric power, used for charging the battery and as a supplementary source for the action of the vehicle.

4) The normal driving - the heat engine is not used mostly; the electricity generation is not necessarily required.

5) During acceleration - during acceleration from the normal speed, the engine speed increases the heat and at the same time the generator starts producing electricity; by using the electric power and the electricity from the battery, the electric motor boosts its power by promoting its accelerated process.

In the papers [3], [4] we have presented other mechanisms that allow the coupling of the power sources, one being the double satellite.

2. The introduction of the functional model

The main component of the functional model is the mechanical system of coupling the power sources.

The kinematic scheme of it is the one from figure 2.

The mechanism used for the coupling of the power sources is a planetary mechanism with two degrees of mobility.

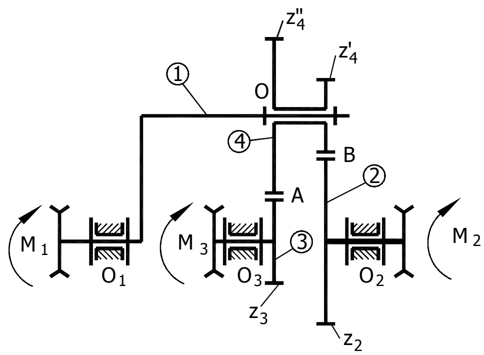

Figure 2. The kinematic coupling mechanism scheme.

The functional mechanism together with the electric engines and an electric engine with permanent magnets used as brake is presented in figure 3.

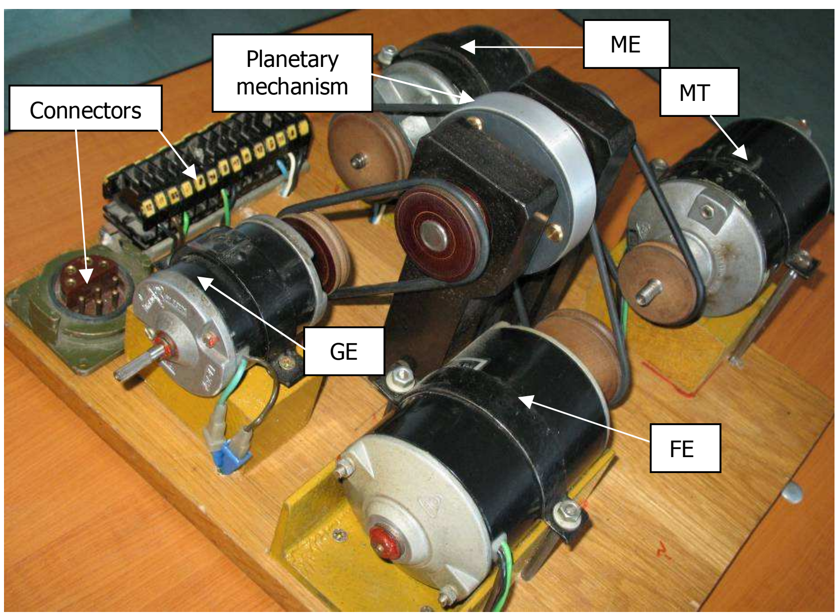

Figure 3. View with the functional mechanism.

Electrical motors with continuous current with a stator with permanent magnets. Were chosen such engines because the external characteristic is practically linear, which allows, depending on load, the approximation of any external feature.

There were chosen three types of electric motors.

For the ME engine was chosen an EA 433 engine, with a nominal speed of 2100 rpm for a supply voltage of 12 V and a useful power of 40 W.

For MT and FE were chosen two electrical engines EA 511, with a nominal speed of 2200 rpm for a supply voltage of 24 V and a useful power of 60 W.

For GE was chosen an EA 244 engine, with a nominal speed of 3000 rpm for a supply voltage of 12V and a useful power of 27 W. Being a functional model and having a didactic purpose, there have been considered the fallowing:

- using a voltage variating device for simulating a vehicle speed;
- for small speeds to operate a single engine;
- for average speeds to operate two electric engines;
- and a third optional engine to be used as a generator;
- a third operating regime "pedal to the bottom" means to supply with maximum voltage all the three engines.

There were used four electrical engines of continuous current, the fourth engine being used as a generator, simulating the resistance encountered by car in movement.

This continuous current engine with permanent magnets has continuously connected a consumer (car lamp) operating as a generator. In the lamp circuit has been installed an ammeter and a voltmeter to estimate the consumed power.

For varying the supply voltage of the engines from zero to 24 V, the serial solution in the primary of a transformer of 220/24 V was adopted for an industrial modulus of voltage variation, with a power of 500 W. Therefore it was avoided the utilisation of an adjustable voltage power sources. The assembly is composed of three relays *Rl*1 with at least three pairs of contacts and *Rl*2, *Rl*3 (auto relay) with a pair of contacts.

The electronic scheme also includes a monophased rectifier module (diode bridge) and an electronic assembling. The electronic device was placed in a metal housing. Inside are the voltage variating device, the three relays, the diodes bridge (auto diode bridge of 35 A), a 24 Vcc power supply and the electronic assembly.

3. Differential equations of motion

The planetary mechanism with planetary carrier is actioned by three couples considered to be resistant or driving.

In this way it can be simulated all modes of functioning of this mechanism when it is used in the coupling the power sources.

It is considered the coupling mechanism from figure 2 wheeled by the moments M_1, M_2, M_3.

There are used the notations:

- $z_2, z_3, z_4^{'}, z_4^{''}$ number of teeth of the gears;

– $\theta_1, \theta_2, \theta_3$ shaft rotation angles 1, 2, 3;
– $\omega_1, \omega_2, \omega_3, \omega_4$ absolute angular speeds of the elements *1*, *2*, *3*;
– J_1, J_2, J_3, J_4 inertia moments of the elements *1*, *2*, *3*;
– i_1, i_2 transmission ratios:

$$i_1 = \frac{z_2}{z_1^{'}}; i = \frac{z_2 z_4^{''}}{z_3 z_4^{'}}; \tag{1}$$

– M_1, M_2, M_3 moments that act elements *1*, *2*, *3*;
– m_4 mass of the element *4*;
– R crank radius 1;
– A, B, C parameters defined by relations:

$$A = J_1 + m_4 R^2 + \left(1 + i_1^2\right) J_4 + (1 - i) J_3;$$
$$B = -(1 - i) i J_3 + J_4 (1 + i_1) i_1; \tag{2}$$
$$C = J_2 + J_3 i^2 + J_4 i_1^2;$$

– $\delta\theta_1, \delta\theta_2, \delta\theta_3$ virtual angular displacements.

From the kinematic analysis result the relations:

$$\omega_3 = \omega_1 (1 - i) + \omega_2 i; \tag{3}$$

$$\omega_4 = \omega_1 (1 + i_1) + \omega_2 i_1. \tag{4}$$

The kinetic energy of the system is written:

$$E_C = \frac{1}{2}\left(A\dot{\theta}_1^2 + 2B\dot{\theta}_1\dot{\theta}_2 + C\dot{\theta}_2^2\right). \tag{5}$$

The virtual mechanical work is:

$$\delta L = M_1 \delta\theta_1 + M_2 \delta\theta_2 + M_3 \delta\theta_3 \tag{6}$$

and as the virtual displacements verify relation (3):

$$\delta\theta_3 = \delta\theta_1 (1 - i) + \delta\theta_2 i \tag{7}$$

it results:

$$\delta L = [M_1 + M_2 (1 - i)]\delta\theta_1 + [M_2 + M_3 i]\delta\theta_2 \tag{8}$$

and from here are obtained the generalized forces:

$$Q_1 = M_1 + M_3 (1 - i);$$
$$Q_2 = M_2 + M_3 i. \tag{9}$$

Appling Lagrange equations there are obtained the next equations:

$$A\ddot{\theta}_1 + B\ddot{\theta}_2 = Q_1; B\ddot{\theta}_1 + C\ddot{\theta}_2 = Q_2 \tag{10}$$

or

$$\dot{\omega}_1 = \frac{[M_1 + M_3(1-i)]C + (M_2 + M_3 i)B}{AC - B^2};$$
$$\dot{\omega}_2 = \frac{[M_1 - M_3(1-i)]B + (M_2 + M_3 i)A}{AC - B^2}. \quad (11)$$

4. Coupling the engines and the continuous current generators

To obtain the differential movement equations there are considered moments with a variation as:

$$M_i = M_{0i} - b_i \omega_i, \quad i = 1,2,3. \quad (12)$$

And equations (11), taking into account relation (3), become:

$$\dot{\omega}_1 = \frac{\left\{M_{01} + (1-i)M_{03} - \omega_1\left[b_1 + (1-i)^2 b_3\right] - i(1-i)b_3\omega_2\right\}C}{AC - B^2} -$$
$$- \frac{\left[M_{02} + iM_{03} - i(1-i)b_3\omega_1 - \omega_2\left(b_2 + i^2 b_3\right)\right]B}{AC - B^2};$$
$$\dot{\omega}_2 = \frac{\left\{M_{01} + (1-i)M_{03} - \omega_1\left[b_1 + (1-i)^2 b_3\right] - i(1-i)b_3\omega_2\right\}B}{AC - B^2} - \quad (13)$$
$$- \frac{\left[M_{02} + iM_{03} - i(1-i)b_3\omega_1 - \omega_2\left(b_2 + i^2 b_3\right)\right]A}{AC - B^2}.$$

If the fallowing notations are used:

$$\tilde{Q}_1 = M_{01} + (1-i)M_{03} - \omega_1\left[b_1 + (1-i)^2 b_3\right] - i(1-i)b_3\omega_2$$
$$\tilde{Q}_2 = M_{02} + iM_{03} - i(1-i)b_3\omega_1 - \omega_2\left(b_2 + i^2 b_3\right) \quad (14)$$

then the system is written:

$$\dot{\omega}_1 = \frac{\tilde{Q}_1 C - \tilde{Q}_2 B}{AC - B^2}; \quad \dot{\omega}_2 = \frac{\tilde{Q}_1 B + \tilde{Q}_2 A}{AC - B^2}. \quad (15)$$

System (2.1.2), using the notations:

$$M_{11} = M_{01} + (1-i)M_{03}; M_{12} = M_{02} + iM_{03}; \quad (16)$$
$$\alpha = b_1 + (1-i)^2 b_3; \beta = i(1-i)b_3; \gamma = b_2 + i^2 b_3 \quad (17)$$

$$a_{01} = \frac{1}{AC - B^2}(M_{11}C + M_{12}B)$$
$$a_{02} = \frac{1}{AC - B^2}(M_{11}B + M_{12}A) \tag{18}$$

$$a_{11} = \frac{1}{AC - B^2}(\alpha C + \beta B); a_{12} = \frac{1}{AC - B^2}(\beta C + \gamma B);$$
$$a_{21} = \frac{1}{AC - B^2}(\alpha B + \beta A); a_{22} = \frac{1}{AC - B^2}(\beta B + \gamma A) \tag{19}$$

$$\{\omega\} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}; \ \{a_0\} = \begin{bmatrix} a_{01} \\ a_{02} \end{bmatrix}; \ [A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \tag{20}$$

is transcribed into the matrix differential equation:

$$\{\dot{\omega}\} = \{a_0\} - [A]\{\omega\}. \tag{21}$$

5. Solution for the general case.

The determinant of the matrix $[A]$, by solving the calculations, after reductions and simplifications, is written:

$$\det[A] = \frac{b_1 b_2 + b_1 b_3 i^2 + b_1 b_3 (1 - i)^2}{AC - B^2} \tag{22}$$

and as $AC - B^2 \neq 0$ it results that it is nonzero, so:

$$b_1 b_2 + b_1 b_3 i^2 + b_1 b_3 (1 - i)^2 \neq 0\,. \tag{23}$$

Condition (23) is generally accomplished if $b_i > 0, i = 1,2,3$ and so matrix $[A]$ is irreversible.

In these conditions, by using the substitution:

$$\{\omega\} = \{\Omega\} + \{\Omega_0\} \tag{24}$$

where:

$$\{\Omega_0\} = [A]^{-1}\{a_0\} \tag{25}$$

is obtained the equation:

$$\{\dot{\Omega}\} = -[A]\{\Omega\}. \tag{26}$$

The eigen values r_1, r_2 of the matrix $[A]$ are generally negative and are derived from the equation:

$$r^2 + (a_{11} + a_{22})r + a_{11}a_{22} - a_{12}a_{21} = 0\,. \tag{27}$$

The modal matrix elements:

$$[\mu]=\begin{bmatrix}1 & 1\\ \mu_1 & \mu_2\end{bmatrix} \tag{28}$$

are obtained from relations:

$$\mu_i=-\frac{r_i+a_{11}}{a_{12}} \tag{29}$$

and the general solution is written:

$$\{\Omega\}=[\mu][E]\{C\}, \tag{30}$$

where:

$$[E]=\begin{bmatrix}e^{r_1t} & 0\\ 0 & e^{r_2t}\end{bmatrix}; \tag{31}$$

$$\{C\}=\begin{bmatrix}C_1\\ C_2\end{bmatrix}. \tag{32}$$

Considering the initial conditions:

$$t=0;\{\omega\}=\{0\}, \tag{33}$$

it is obtained from (24) condition:

$$t=0;\{\Omega\}=-\{\Omega_0\} \tag{34}$$

and from (30) it results:

$$\{C\}=-[\mu]_{-1}\{\Omega_0\}. \tag{35}$$

As a consequence there are obtained the matrix solutions:

$$\{\Omega\}=-[\mu][\varepsilon][\mu]^{-1}\{\Omega_0\}; \tag{36}$$

$$\{\omega\}=\left[[I]-[\mu][\varepsilon][\mu]^{-1}\right]\{\Omega_0\}; \tag{37}$$

respectively the scalar solutions:

$$\begin{aligned}\omega_1&=\Omega_{01}-\frac{1}{\mu_2-\mu_1}\left[\left(\mu_2e^{r_1t}-\mu_1e^{r_2t}\right)\Omega_{01}+\left(-e^{r_1t}+e^{r_2t}\right)\Omega_{02}\right];\\ \omega_2&=\Omega_{02}-\frac{1}{\mu_2-\mu_1}\left[\mu_1\mu_2\left(e^{r_1t}-e^{r_2t}\right)\Omega_{02}+\left(-\mu_1e^{r_1t}+\mu_2e^{r_2t}\right)\Omega_{02}\right].\end{aligned} \tag{38}$$

6. Solution in stationary regime

In stationary regime $(t\to\infty;e^{r_1t}\to 0;e^{r_2t}\to 0)$ there are obtained:

$$\omega_1=\Omega_{01};\omega_2=\Omega_{02}. \tag{39}$$

Values Ω_{01},Ω_{02} are obtained from relation (26), respectively from the system:

$$Q_1 C + Q_2 B = 0;$$
$$Q_1 B + Q_2 A = 0; \tag{40}$$

where Q_1, Q_2 are obtained from relations (3) in which are replaced ω_1, ω_2 cu Ω_1, Ω_2.

System (40) is a homogeneous system in Q_1, Q_2 and as the determinant of the system is $AC - B^2 \neq 0$ it results that the system admits only one real solution:

$$Q_1 = 0; Q_2 = 0\,, \tag{41}$$

meaning:

$$M_{01} + (1-i)M_{03} = [b_1 + (1-i)^2 b_3]\Omega_{01} + i(1-i)b_3\Omega_{02};$$
$$M_{02} + iM_{03} = i(1+i)b_3\Omega_{01} + (b_2 + i^2 b_3)\Omega_{02}. \tag{42}$$

It is obtained the solution in stationary phase:

$$\Omega_{01} = \frac{M_{01}(b_2 + i^2 b_3) - M_{02} i(1-i)b_3 + M_{03}(1-i)b_2}{b_1 b_2 + b_1 b_3 i^2 + b_2 b_3 (1-i)^2} \,|\, 1-i\,;$$
$$\Omega_{02} = \frac{-M_{01} i(1-i)b_3 + M_{02}[b_1 + (1-i)^2 b_3] + M_{03} i b_1}{b_1 b_2 + b_1 b_3 i^2 + b_2 b_3 (1-i)^2} \,|\, i\,; \tag{43}$$
$$\Omega_{03} = \Omega_{01}(1-i) + \Omega_{02} i = \frac{M_{01} b_2 (1-i) + M_{02} b_1 i + M_{03}[b_1 i^2 + b_2 (1-i)^2]}{b_1 b_2 + b_1 b_3 i^2 + b_2 b_3 (1-i)^2}.$$

7. Conclusions

It appears that the solution in phase regime depends on the engine's characteristics and on the characteristics A, B, C of the system, while under permanent regime, the solution depends only on the $M_{01}, b_i, i = 1,2,3$ characteristics of the engines. To fall into the work range of the electric motors the solutions (43) means that the fallowing conditions must be respected:

$$M_{01} - b_1\Omega_{01} > 0;\ M_{02} - b_{21}\Omega_{02} > 0;\ M_{03} - b_3\Omega_{03} > 0. \tag{44}$$

If the notations below are used

$$\tilde{\Omega}_1 = \frac{M_{01}}{b_1};\ \tilde{\Omega}_2 = \frac{M_{02}}{b_2};\ \tilde{\Omega}_3 = \frac{M_{03}}{b_3}\,, \tag{45}$$

then the conditions (44) are written with the fallowing forms

$$(1-i)[\tilde{\Omega}_1(1-i)+\tilde{\Omega}_2 i-\tilde{\Omega}_3]>0;$$
$$i[\tilde{\Omega}_1(1-i)+\tilde{\Omega}_2 i-\tilde{\Omega}_3]>0; \quad (46)$$
$$(-1)[\tilde{\Omega}_1(1-i)+\tilde{\Omega}_2 i-\tilde{\Omega}_3]>0.$$

The inequalities (46) are incompatible simultaneously which indicates us that the system does not allow the simultaneous engagement of three power sources.

This means that at least one of the power sources must be a consumer of the generator type ($M_{01}<0$) or of brake type $\left(b_i=0; M_{0i}<0\right)$.

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