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Analysis of a Mechanical System's Dynamic Properties by Vibrations Measurements

This paper aims to present some theoretical notions about the solution of the reverse problem in the dynamic response study of a mechanical system. Thus, by measuring vibration, some dynamic properties of the mechanical system considered can be determined.

Keywords*: mass, stiffness, damping, measuring vibration*

1. Introduction

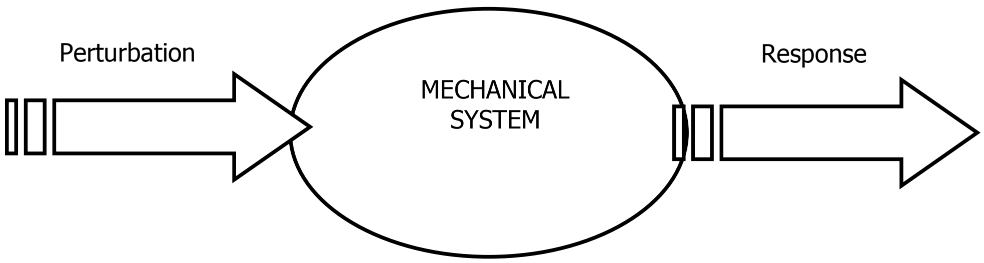
Whatever vibration phenomena we encounter around us (some characteristic of living beings, other technical based), basic laws by which they take place and their mathematical formulations are the same.

For the dynamic study of a mechanical system subject to vibration, the following steps are needed generally [1]:

- Defining the problem, (i.e. system description to be reviewed);
- Physical modelling, (i.e. creating a model close to real) - but simpler – therefore easier to analyze;
- Mathematical modelling – writing the physical model motion equations;
- Mathematical model dynamic study, (i.e. solving the motion equations);
- Comparison of the physical model to the real one, to verify the correctness of the model.

Solving a vibration problem is, actually, determining relations between the perturbations, the system response and its dynamic characteristics.

If for the direct problem, when known the equations describing the dynamic behaviour of the system, it is required to determine the system response (for example, the equations describing the dynamics of a crane are known and the response to the shock produced by a strong gust of wind is required), for the reverse problem, the disturbance is required – when the system is known or the motion equations of the mechanical system are required when the response is known.



2. Problem definition

If we consider a mechanical system consisting of a rigid body and supports (springs and dampers – assumed to be linear), the reverse problem consists in determining the dynamic properties (i.e. the gravity centre, the inertia moment, the springs' stiffness or the dampers' coefficients), using data obtained as a result of vibration measurement.[2]

For this analysis we consider a rigid body mass *m*, which is supported in *p* points, by compression and torsion springs, as well as compression and torsion dampers (all are linear).

This can be schematically represented as in Figure 1:

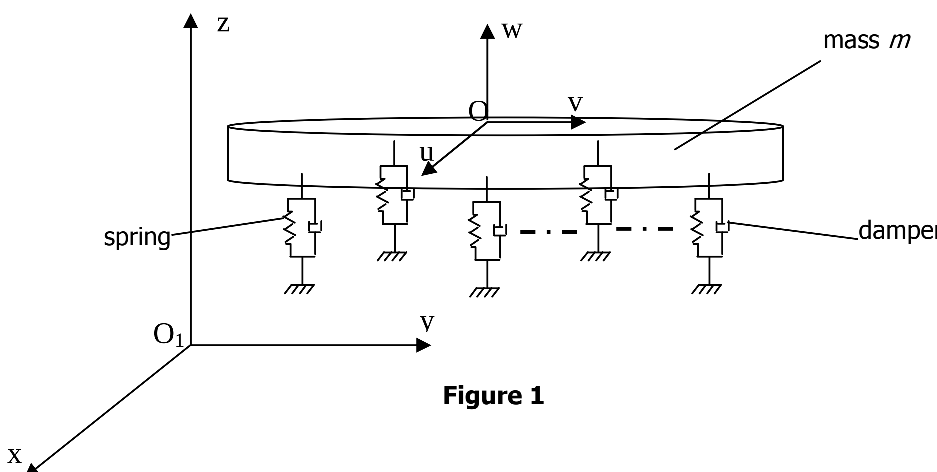


Figure 1

Also we will consider two reference systems: one on the ground (O_1xyz), and the other attached to the rigid body (*Ouvw*).

If x, y, z are the translational displacements and $\varphi_x, \varphi_y, \varphi_z$ are the rotational displacements around axes O_{1x}, O_{1y}, O_{1z}, then, the displacement vector of the gravity centre and the mass matrix in rapport to the centre of gravity will have the following expressions:

$$D_g = (x, y, z, \varphi_x, \varphi_y, \varphi_z) \tag{1}$$

$$M_g = \begin{pmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x & -J_{xy} & -J_{xz} \\ 0 & 0 & 0 & -J_{yx} & J_y & -J_{yz} \\ 0 & 0 & 0 & -J_{zx} & -J_{zy} & J_z \end{pmatrix} \tag{2}$$

Stiffness coefficients matrix corresponding to spring i will be written as:

$$K_i = \begin{pmatrix} k_{ixx} & -k_{ixy} & -k_{ixz} & 0 & 0 & 0 \\ -k_{iyx} & k_{iyy} & -k_{iyz} & 0 & 0 & 0 \\ -k_{izx} & -k_{izy} & k_{izz} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{ixx}^T & -k_{ixy}^T & -k_{ixz}^T \\ 0 & 0 & 0 & -k_{iyx}^T & k_{iyy}^T & -k_{iyz}^T \\ 0 & 0 & 0 & -k_{izx}^T & -k_{izy}^T & k_{izz}^T \end{pmatrix} \tag{3}$$

k and k^T as the stretching and torsional stiffness coefficients.

Being a rigid solid body, moving its gravity centre motion can be completely determined when knows any point of its movement is known. Thus, under [2]:

$$D_g = G_{gi} \cdot D_i \tag{4}$$

where,

$$G_{gi} = \begin{pmatrix} 1 & 0 & 0 & 0 & w_i - w_g & v_g - v_i \\ 0 & 1 & 0 & w_g - w_i & 0 & u_i - u_g \\ 0 & 0 & 1 & v_i - v_g & u_g - u_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (5)$$

and represents the coordinate transformation matrix from the point i to the point g.

$$D_i = (x_i, y_i, z_i, \varphi_x, \varphi_y, \varphi_z) \qquad \text{(displacement vector of the point } i\text{)} \qquad (6)$$

and an analyzed rigid solid body stiffness, in relation to the gravity centre, can be written as:

$$K_g = \sum_{i=1}^{p} G_{ig}^T \cdot K_i \cdot G_{ig} \qquad (7)$$

where G_{ig} is the inverse of matrix G_{gi}. [2]

Similarly to the determination of matrix K_g, the damping matrix C_g can be expressed, using the damping coefficients matrix corresponding damper i:

$$C_i = \begin{pmatrix} c_{ixx} & -c_{ixy} & -c_{ixz} & 0 & 0 & 0 \\ -c_{iyx} & c_{iyy} & -c_{iyz} & 0 & 0 & 0 \\ -c_{izx} & -c_{izy} & c_{izz} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{ixx}^T & -c_{ixy}^T & -c_{ixz}^T \\ 0 & 0 & 0 & -c_{iyx}^T & c_{iyy}^T & -c_{iyz}^T \\ 0 & 0 & 0 & -c_{izx}^T & -c_{izy}^T & c_{izz}^T \end{pmatrix} \qquad (8)$$

The gravity centre motion equation is defined as:

$$M_g \cdot \ddot{D}_g + C_g \cdot \dot{D}_g + K_g \cdot D_g = F_g \qquad (9)$$

where,

$Dg, \dot{D}g, \ddot{D}g$ are vectors of displacement, velocity and acceleration of the gravity center;

F_g is the amount of force acting on gravity center (resultant of all forces acting on the body).

Because the gravity centre position is not known, the displacements measured at measurement points must be expressed in relation to a known point on the rigid solid body. If this point is A, the equation of motion of point A will be written as:

$$M_A \cdot \ddot{D}_A + C_A \cdot \dot{D}_A + K_A \cdot D_A = F_A \quad (10)$$

Using notation:

$$M_A^{-1} \cdot K_A = Q_A \quad (11)$$

According to [2] and [3]:

$$M_g \cdot G_{gA} \cdot Q_A = K_g \cdot G_{gA} \quad (12)$$

where G_{gA} is defined similar (5).

Also, the total mass of bodies that make up the mechanical system, can be easily determined in most cases. This fact greatly simplifies calculations.

Exploiting the equations (4) and (12), the coordinates of the gravity centre of rigid solid body can be determined, as well as the spring stiffness coefficients and the dampers damping coefficients.

3. Conclusions

The dynamic properties of a mechanical system are now determined by two methods:

- The analytical method;
- The experimental method.

If we are dealing with complex structures - in situ - the method briefly presented in these pages is very attractive, other methods are inaccurate, time consuming and difficult.

This method can be successfully used to track how a structure deteriorates in the course of time (for example a bridge).

In a future article, the authors are proposing to deepen and extend the calculations presented in this paper, including a multiple structure of rigid bodies.

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