ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REȘIȚA
ANUL XIII, NR. 1, 2006, ISSN 1453 - 7397

Iancu Tătucu, Lenuţa Suciu

On Simulation of Electromagnetic Devices with Standard Coils

For many electromagnetic devices, the field coils may be considered as being stranded, i.e. we may neglect the surface effect. We thus consider such a coil with N turns (windings), the transversal section of one turn S_c *, the space factor* σ *, supplied with electric voltage* U_r *. We term* S_b *the sectional area of the coil, but many simulation software programmes are conceived to admit only massive electric conductors supplied in voltage. Consequently, it is imposed that a massive conductor replaces the coil, with the same geometry as the coil and having the cross section equal to* S_s *In order to ensure, that the electromagnetic process takes place in a way similar to the case of the real coil, the power supply of the conductor should be made with a reduced voltage. The paper establishes the connection relation between these voltages according to the parameter, which we desire to keep unchanged.*

1. Introduction

Most of electromagnetic devices used in industrial practice have standard coils, with a certain number of spires.

But many simulation programmes admit only excitation coils, made of a single massive conductor. Consequently, it is necessary to replace the real, standard coil, with a single massive conductor, having the same geometry as the standard coil.

The purpose of this paper is to establish the connection relation between the two electric voltages.

Obviously, so that this replacement should not modify the studied electromagnetic process, the massive conductor must be supplied with a lower voltage than the supply voltage of the real coil.

2. The replacement of the real coil with a massive conductor

We consider an electromagnetic device having an excitation coil, constituted from an N number of spires, and made of an electric conductor with the S_c surface of the cross section.

In general, we understand by the cross section of a coil what we obtain by means of intersecting the coil with a plane normal (perpendicular) on the electric current lines. With a fair enough approximation we can admit that the current lines are disposed in planes perpendicular on the symmetry axis of the coil, meaning that we can admit that the cross section is the very axial section of he coil, i.e. its intersection with a plane passing through the coil symmetry axis.

Besides, no great errors will intervene if we start from the hypothesis that the current lines are disposed in planes normal on the symmetry axis for any line of flow (current).

Based on this hypothesis, we consider that the line of flow of a source is that passing through the middle of the cross section of the spire. Considering all the above, it follows that such a line is placed in a plane perpendicular on the coil symmetry axis. Is also a result that in this plane we find the lines of flow of all the spires from a coil layer. The intersection of the coil with the respective plane will be called the section of a coil layer, represented in fig.1.a.

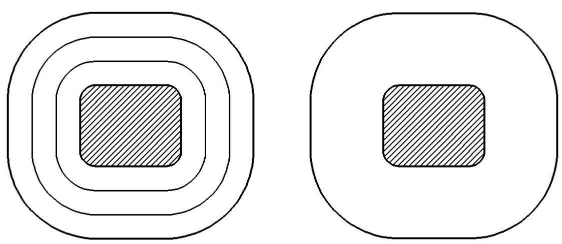
a) b)

Figure 1. Perpendicular section of a layer of the real section (a) and for the massive conductor (b).

Let S_b be the surface of the axial section of the coil. Because of the casing on which the coil is placed and because of the insulation between the spires, they do not occupy the entire cross section of the coil, they cover only a part resulting form the expression of the block coefficient σ defined through the relation:

$$\sigma = \frac{NS_c}{S_b} \qquad (1)$$

The coil is replaced with a massive conductor having the same shape and dimensions as the real coil. It means that the real coil is replaced with a single spire with the S_b surface of the cross section, and the normal section has the same shape and dimensions as the real coil, as shown in fig.1.b.

Considering the above hypotheses for such an one-spire coil, we may consider as line of flow the line passing through the middle of the axial section, which is the very middle line of the normal section of fig.1.b. and let L be its length.

We also adopt the simplifying hypothesis that the lines of flow of all N spires of the real coil are identical with the middle line above.

Consequently, if L_b is the line of flow of the real coil, then:

$$L_b = N \cdot L \qquad (2)$$

Admitting that the current density has the same value J in any point of each spire of the real coil, and applying the definition relation of the electric voltage along the lines of flow in spires, we obtain:

$$U = N \rho J \ell , \qquad (3)$$

where we have considered the mentioned hypotheses and relation (2), and the notations unmentioned yet have the following meanings: U – the electric supply voltage of the real coil, ρ - electric receptivity of the spires material.

The same way and under the same simplifying hypotheses for the massive conductor we obtain:

$$U_r = \rho J^{'} \ell , \qquad (4)$$

where:

U_r – electric supply voltage of the massive conductor; $J^{'}$ – current density in a random point of the massive conductor.

The connecting relation between the two current densities depends upon the aspect of the studied electromagnetic process, which is desirably the same for both cases.

Thus, if we impose the same specific caloric power on the volume unit corresponding to the J effect of the electric current, then J^{I}=J and from relations (4) and (3) we get:

$$U_r = \frac{U}{N} \qquad (5)$$

So, if the massive conductor is supplied with and electric voltage N times smaller that the electric supply voltage of the real coil, then the current density is the same as in the case of the real coil.

Similarly, if by running the programme for the case when the coil is replaced with the massive conductor, a specific caloric power of the value p is obtained, this value is the same as in the case of the real coil.

Obviously the caloric power specific to the entire volume is different. Considering that the specific caloric power p is uniformly distributed in both cases and adopting the notations P, P_b, the caloric power in the two cases, by definition:

$$P = \rho J^2 S_b \ell \quad , \quad P_b = \rho J^2 N S_c \ell \qquad (6)$$

After running the programme we obtain P and in order to deduce the expression of P_b, for relations (6) and (1) it results:

$$P_b = \sigma P \tag{7}$$

Thus the caloric power developed in the volume of the real coil is obtained through multiplying the caloric power obtained from the programme running by the block coefficient.

Let I, I_b be the current amperage through the massive conductor, and through the spires of the standard coil, respectively, then, by definition:

$$I = J S_b \ , \ I_b = J S_c \tag{8}$$

From relations (8) and (1) we obtain:

$$I_b = \frac{\sigma}{N} \cdot I \tag{9}$$

So, the electric current amperage in the real coil is obtained through multiplying the current amperage obtained form the programme running by the σ / N ratio.

In many practical applications we consider only the magnetic aspect of the studied process, such as: the distribution of the magnetic induction in the considered electromagnetic device, the forces that act on its mobile armature, etc. So that these quantities be the same in both situation we must impose the condition that: in the device range, the magnetic field be the same, which is realised if the solenations, are equal, i.e.:

$$\theta_b = \theta \ , \tag{10}$$

where: θ_b is the solenation of the standard coil; θ is the solenation of the massive coil.

Taking into account the definition of a coil solenation, relation (10) becomes:

$$N I_b = I \tag{11}$$

Condition (11) can be fulfilled only if the current density in the case of the massive coil has another value J^I, different form the value J obtained in the case of the standard coil. Consequently, from relations (11) and (8) it results:

$$N J S_c = J^I S_b \ , \tag{12}$$

from which, based on relation (1) we get:

$$J = \frac{J^I}{\sigma} \tag{13}$$

From relation (13) it follows that in the case of the massive coil we must obtain a current density $J^I = \sigma J$, which is possible only if we modify that supply voltage of the massive coil, that should have a value also noted U_r

If in relation (4) we replace J^I by σJ and keep in mind relation (3) we arrive at:

$$U_r = \frac{\sigma}{N} U \tag{14}$$

Thus, if we replace the real coil with a massive conductor, so that in the range of the studied device we should obtain the same magnetic field as in the case of the real coil, the massive coil must be supplied with an electric voltage obtained through multiplying the electric supply voltage of the device by σ / N.

It is obvious that as a result the quantities p_b, P_b, I_b will have different values compared to the values obtained after the running of the programme, which were p, P, I. In order to arrive at the connection relations in the expressions of the quantities p, P, I from relations (6) and (8) we replace J by $J^I = \sigma J$, and consequently:

$$p_b = \frac{1}{\sigma^2} p \tag{15}$$

$$P_b = \frac{1}{\sigma} p \tag{16}$$

$$I_b = \frac{1}{N} \cdot I \tag{17}$$

With the values p, P, I determined, their real values, noted with p_b, P_b, I_b respectively are calculated from relations (15), (16), (17).

3. Supply of the excitation coils made of segments

In order to simulate the functioning of an electromagnetic device we should first introduce the value of a quantity for each coil, operation called the supply of the respective coil.

In general there are three supply modalities: a) introducing the value J of the current density in the points of the coil; b) introducing the value I_b of the current amperage of the coil, this modality is also called current supply of the coil; c) imposing the value U of the voltage drop on the coil and thus talking about the voltage supply of the coil.

In the first case, we usually do not directly measure the value of J, but the value f the amperage I_b, and the current density is calculated from the relation:

$$J = \frac{N I_b}{S_b} \tag{18}$$

For the results obtained to be correct we must also comply with the Coulomb's sampling condition, meaning that it is necessary that the current density should have a solenoidal character. But if the coil has curved portions, this condition is not met and the results will be affected by errors. For this reason and considering also that actually, according to relation (18), the current density may be determined only I_b is known, the first supply modality is very rarely used and only for a stationary regime of processes taking place in the studied device.

Consequently, supply modes b and c are more frequently used. Moreover, any simulation programme of electromagnetic processes is based on the mathematical model of Maxwell's equations or on that of the magnetic potential vector, and for both we must know the current density J.

In the case of the current supply mode (b) the current amperage I_b is introduced for an axial section of the coil and in order to impose that the current density vector $\overline{J}$ be perpendicular on the respective section, we set the condition that the electric potential should have the same value in all points of the section. We say that the knots from the respective section have been coupled in a degree of liberty equal to the electric potential. Knowing N, S_b and imposing the value of I_b the programme determines the value J of the current density from relation (18).

For the voltage supply (mode c), if V is the electric potential in a random point of it, the current density vector in the respective point is obtained from relation:

$$\overline{J} = -\frac{1}{\rho}\nabla V \,, \qquad (19)$$

where ∇ is the gradient differential ratio.

In order to impose the condition that the current amperage should have the same value I_b in any section of each spire, all the coil knots are coupled in the "current" degree of liberty, without introducing its value.

Evidently relation (19) may be used only if we know the spatial distribution of the electric potential in the coil volume, that is knowing the values of the electric potential in as many points as possible or knowing the electric potential as depending on the co-ordinates of a random point form the coil volume. But for most of the practical applications this distribution is not known, and , as we specified for the (c) supply mode, we know the value U of the voltage drop on the coil.

If we deal with a standard coil, in some simulation programmes we first define on each portion of this coil the direction of the $\overline{J}$ vector by introducing the values of the components of a vector having an arbitrary length, for instance, equal to the measuring unit, along the direction of the axes of a global co-ordinates system, specific to the programme, or of a local system of co-ordinates defined by the user. The local system of co-ordinates may be: Cartesian, cylindrical or spherical. For instance, if for a portion we introduce the vector (1,0,0), and then the vector $\overline{J}$ has the direction of the x-axis of the system of co-ordinates defined for the respective portion.

In all simulation programmes the range of the studied device is divided into an n number of disjoint zones, called finite elements, and the mathematical model used is solved for each and every finite element.

Also, relation (19) is used in order to determine the value of the $\overline{J}$ vector from the domain occupied by each finite element.

In this purpose, the value introduced, is distributed equally to all the n finite elements, which means that for each finite element the electric voltage drop has the value U/n.

Further on, for each finite element a problem of electric conduction is solved, considering that it is subjected to a t electric potential difference equal to the above.-mentioned value. The result is constituted by the values of the electric potential V , in the knots of each finite element, which are used for solving equation (19)through a numerical method, thus obtaining the vector $\overline{J}$ for each finite element.

The authors of the present paper found that, when portions of different shapes, make the coil for instance similar to those in fig.1.a, the results obtained are totally erroneous. Thus in the case of n electric transformer we have measured the current intensity through the coil and knowing the surface of the cross section of each spire we have computed the value of the current density. Through simulation of the electric transformer with specialised software, we obtained, for the current density, values that are approximately 100 times higher than the calculated value.

The apparition of such huge errors is explained through the fact that, as specified above, the voltage drop is distributed equally to all the finite elements in the coil volume. If the mesh is automatically realised by the simulation programme, then in the curved portions of the coil the density of finite elements is way higher than in the rectilinious portions. It means that in the curves portions we witness a voltage drop which is much higher than that attributed to the rectilinious portions, even if the curved parts have an average length much smaller than those of the rectilinious ones. In order to reduce these errors, within this paper we forward the hypothesis that to each portion is allotted a voltage drop directly proportional with the average length of the normal section of that portion.

Thus the coil is divided in segments so that each of them should have a regular geometrical shape. For instance, the coil in fig.1.a is divided into eight segments, four rectilinious parallelipipedical and four zones having the shape of a quarter of a hollow cylinder.

The whole coil is allotted the voltage drop U, given by relation (3). If V_b is the coil volume, then relation (3) becomes:

$$U = N \rho J \frac{V_b}{S_b} \qquad (20)$$

Starting form the hypothesis that the coil has the same cross section of surface S_b, if R is a random segment, it receives a voltage drop:

$$U_R = N \rho J \frac{V_{bR}}{S_b}, \qquad (21)$$

where V_{bR} is the volume of the portion considered.

From relations (21) and (20) we obtain:

$$U_R = \frac{V_{bR}}{V_b} U \qquad (22)$$

Generally, the very simulation programme allows the calculation of both volume V_b of the coil and the volumes V_{bR} of the portions, and with relation (2) we calculate the voltage drops U_R received by them.

For each segment of the coil we introduced a voltage drop equal to the calculated value U_R, but the coupling in the "current" liberty degree was made for the knots of the entire coil.

We obtained results that were very close to the calculated or measured values.

4. Conclusion

The relations established in the paper allow that in the simulation of an electromagnetic device massive conductors should replace the standard coils. Thus we can use simulation programmes conceived only for massive conductors, in the simulation of electromagnetic devices with coils made of a certain number of spires.

Moreover, we thus take into consideration also the skin effect neglected by definition in the case of standard coils.

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Addresses:

- Conf. Dr. Eng. Iancu Tătucu, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, i.tatucu@uem.ro
- Assist. Eng. Lenuţa Suciu, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, ilesuciu@uem.ro