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## **Contributions of Determining the Forces Distribution in the Harmonic Drive**

*The paper presents an original method for determining the forces distributions on the portant elements of the harmonic drives. This method is based on the thin wall shells theory anal on the rotating character of the distributed forces that work upon the flexible toothed wheel.*

### **1. Introduction**

In the last 3 - 4 decades, researchers paid more attention to mechanical transmissions through engaging due to multiple advantages that this type of drive has looking to improve them as much as possible but not sufficient enough.

Due to that reason, a series of new mechanical drives appeared which drives from universal joint – drive whit one or two central surge wheels one being the harmonic gear drive [1], [2], [3], [4].

The harmonic gear drive is essential different from the classic mechanic transmissions because it transmits the rotation motion from the driving element to the driven element by elastic deformations (waves) which swings on a flexible element with certain frequency fallowing on harmonic law.

Multiple engaging, particular to harmonic gear drive, generated a iterative method of force distribution which must submit to unilateral elastic connections that take place during the engagement and to the rotating character of the forces system. The fact that the toothed harmonic drive is not so widely used is caused mainly by the limited duration the weakest element – the flexible toothed wheel due to the lack of a study regarding the dynamic behaviour of this, especially regarding the dynamic load.

An important contribution in solving this problem have papers [2], which permitted the identification on the distort forces from the waves generator with disc and came upon the flexible toothed wheel as well as the forces from the harmonic engagement.

The analysis of the theoretical diagrams and that of those obtained through experiments permit to conclude that the diagram of forces from the harmonic

toothed drives are symmetric with the diametral axes of the waves generator and the diagram of distort forces have a cosinusoidal character.

## 2. The distribution of forces in the harmonic toothed drive

Based on the analysis, a method of analytical determining the dynamic load in harmonic toothed drive for generators with cam on elastic bearings is presented.

In transversal plan, upon the flexible toothed wheel (RF) activate the distort forces (P) from the waves generator positioned from the centre of the wheel in radial direction, the radial components of the engaging forces (F) and the tangential elements (which position themselves on the rim of the wheel) determined due to the driven moment of torsion. Experiments showed that behind the crown gear the deformation and shifting of flexible toothed wheel take place, that's why in the dynamic study the area of the flexible toothed wheel will be analysed.

The diagram of the position of forces is shown in Figure 1,a. Curve 1 represents the diagram of the position on distort forces in waves generator on the flexible toothed wheel. Experimentally [2] it was obtained for the distort forces in the case of waves generator with cam and 2 wave of distortion, on cosinusoidal profile:

$$P_i = \frac{w_0 \cdot E \cdot I}{r_2^3} \cdot \left( 0,94 + \frac{103}{z} \right) \cdot \cos 2\varphi_i, \quad (1)$$

where:  $w_0$  is the maximum radial deformation of the flexible toothed wheel;

$I$  - moment of inertness of transversal section of the flexible;  $0 \leq \varphi_i \leq 45^\circ$ ;

$r_2$  - the radius of flexible toothed wheel;

$z$  - the number of rotating elements of the elastic roller.

Curve 2 shows the diagram of position load between the teeth in the peak of the waves area. The diagram of loading is distorted regarding the large axis of the waves generator with angle  $\gamma$  because in the process of harmonic engaging of the teeth the friction forces take place as well.

For the teeth  $i$  ( $i = 1, s$ ), after [2] the loading is determined like:

$$F_i = \frac{F}{n_u \cdot c \cdot i_3^{12}} \cdot \sin \frac{2\pi(i-1)}{(a_1 + a_2) \cdot i_{12}^3}; \quad F = 2M_2 / d_2, \quad (2)$$

where:  $F$  is the tangential force at flexible toothed wheel;

$M_2$  - the moment of torsion;

$i_{12}^3$  - transmission ratio of harmonic drive when waves generator is a driving element;

$d_2$  - dividing diameter of flexible gear pinion;

$n_u$  - number of waves of the waves generator;

$a_0, a_1, a_2, c$  - coefficients chosen from [2], due to the type of waves generator;

$s$  - maximal number of teeth which drive load.

For determining the intensity of force which actuate on the flexible toothed wheel in harmonic engaging area, we project the load on the teeth on the direction of the normal axis to the flexible toothed wheel, in point of contact  $i$  of the waves generator with the flexible toothed wheel (Figure 1,b):

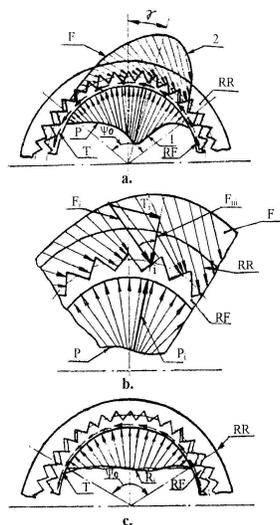
$$F_{ni} = F_i \sin \alpha_i \quad (3)$$

The resultant forces which actuate on point  $i$  upon the flexible toothed wheel in determined as algebraic sum of distort forces  $P_i$  and engaging forces  $F_{ni}$ :

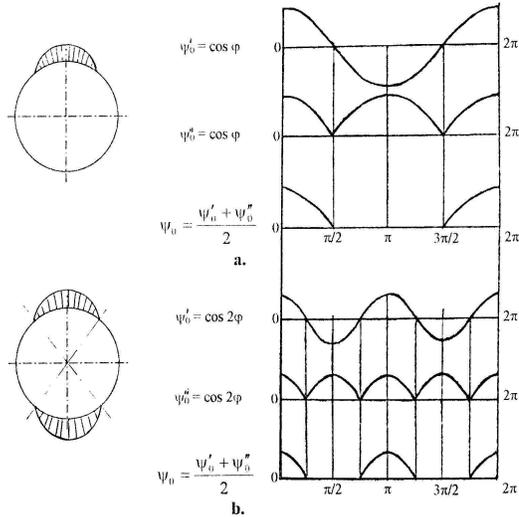
$$R_i = P_i - F_{ni} \quad (4)$$

Total load in obtained by algebraic adding of forces  $R_i$  from the area of action of waves generator:

$$R = \sum_{\varphi_1}^{\varphi_2} R_i \cos \varphi_1 \quad (5)$$



**Figure 1.** The scheme of loading elements in harmonic drive



**Figure 2.** The diagram of distortion that actuate on flexible toothed wheel  
a – one wave; b – two waves

The intensity of reported loading on the length of the spring for two waves drive in determined from integral correlation:

$$\frac{R}{b \cdot l} = 2 \int_0^{\pi/4} q \cdot r_2 \cdot \cos \varphi \cdot d\varphi = 1,4 \cdot q \cdot r_2; \Rightarrow q = R / k r_2 b \quad (6)$$

where:  $k$  – raport coefficient ( $k = 1,42$  for two waves harmonic;  $k = 2$  for one wave harmonic).

In the functional process of harmonic drive the real load, in the limits of every wave can be modified in time with a certain frequency that depends on the toothed frequency, teeth interference and error of circumferential pace. In this case the load intensity must be taken as:

$$q = \frac{R \cdot \delta(n)}{k \cdot r_2 \cdot b} \left\{ \frac{1}{2} \left[ \cos n_u \left( \frac{y}{r_2} + \omega_1 \cdot t \right) + \cos n_u \left( \frac{y}{r_2} + \omega_1 \cdot t \right) \right] \right\} \cos \Omega t \quad (7)$$

where:  $\Omega_t$  the frequency of swings in the limits of every semi-wave depending on above mentioned factors;

$\omega_1$  - angular speed of the waves generator.

### 3. Conclusions

In the thesis was presented a dynamic analyses of toothed harmonic drive based on thin wall shells theory and on rotating character of distributed forces that actuate on flexible toothed wheel.

The analytical dependences were determined for describing the rotating character of disturbing forces system from harmonic drive by using Dirac function.

Allowing the load calculated as (9) or (10) and knowing the shape of distorting of the flexible toothed wheel the problem of the dynamic of flexible toothed wheel can be solved.

### References

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