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Florin Constantin Berinde, Gilbert-Rainer Gillich

Considerations Regarding the Use of the Time-Frequency Representations in Analysis of Vibrations

Time-frequency representations are modern tools in analyzing various signals, including that obtained by measurements of vibrations. A great number of this kind of tools are available, it is the researcher's expertise which lead to the use of the proper one. The authors, based on their expertise in handling data acquisition and analysis trough time-frequency representations, present in the paper considerations regarding the use of this tools, in connection with the operating system's performances.

1. About time-frequency representations

Monitoring of vibrations produced by industrial sources aims to gain information about the sources behavior on one side, and to identify changes in physical properties of the possible damaged structures (such as loss of stiffness) on the other side. Tools for evaluation of signals are numerous. The Fourier Transform is often used because it contains information regarding frequency content, but it can not resolve the exact start of changes for these frequencies. Spectrograms are better able to resolve temporal evolution of frequency content. An accurate analysis allow time-frequency transformations like the Wigner-Ville Distribution, which permit instantaneous frequency estimation at each data point, for temporal resolution of fractions of a second. Variants of the Wigner-Ville Distribution obtained by smoothing in time or frequency (like the Pseudo Wigner-Ville Distribution or the Smoothed Pseudo Wigner-Ville Distribution) are often used because they are more precise, but by smoothing they lose on properties [1]. There are a lot of other time-frequency representations, but we will focus on the spectrogram and the Wigner-Ville Distribution family.

The authors have rich expertise in using time-frequency representations in evaluation of signals, especially vibrations. Some considerations regarding the use of this kind of representations are presented in this paper.

2. Collecting information on time, frequency and phase

First of all, the representations are used for collecting information about time, frequency or phase. The first and second order moments, in time and in frequency, of a time-frequency energy distribution *TFR* are defined as

$$f_m(t) = \frac{\int_{-\infty}^{+\infty} f \cdot TFR(t,f)df}{\int_{-\infty}^{+\infty} TFR(t,f)df} \quad (1)$$

$$B^2(t) = \frac{\int_{-\infty}^{+\infty} f^2 \cdot TFR(t,f)df}{\int_{-\infty}^{+\infty} TFR(t,f)df} - f_m(t)^2 \quad (2)$$

for the *time moments*, and as

$$t_m(f) = \frac{\int_{-\infty}^{+\infty} t \cdot TFR(t,f)dt}{\int_{-\infty}^{+\infty} TFR(t,f)dt} \quad (3)$$

$$T^2(f) = \frac{\int_{-\infty}^{+\infty} t^2 \cdot TFR(t,f)dt}{\int_{-\infty}^{+\infty} TFR(t,f)dt} - t_m(f)^2 \quad (4)$$

for the *frequency moments*. They describe the averaged positions and spreads in time and in frequency of the signal. For some particular distributions, if the signal is considered in its analytic form, the first order moment in time also corresponds to the instantaneous frequency, and the first order moment in frequency to the group delay of the signal. It can also be interesting to consider the *marginal distributions* of a time-frequency representation. These marginals are defined as:

$$m_f(t) = \int_{-\infty}^{+\infty} TFR(t,f)df \quad \textit{time marginal} \quad (5)$$

$$m_t(f) = \int_{-\infty}^{+\infty} TFR(t,f)dt \quad \textit{frequency marginal} \quad (6)$$

and express, by integrating the representation along one variable, the repartition of the energy along the other variable.

A natural constraint for a time-frequency distribution is that the time marginal corresponds to the instantaneous power of the signal, and that the frequency marginal corresponds to the energy spectral density:

$$m_f(t) = |x(t)|^2 \text{ and } m_t(f) = |X(f)|^2 \qquad (7)$$

The interference terms present in any quadratic time-frequency representation, even if they disturb the readability of the representation, contain some information about the analyzed signal. The precise knowledge of their structure and construction rule is useful to interpret the information that they contain. For instance, the interference terms contain some information about the phase of a signal. Let us consider the Pseudo Wigner-Ville Distribution of the superposition of two constant frequency modulations, with a phase shift between the two sinusoids.

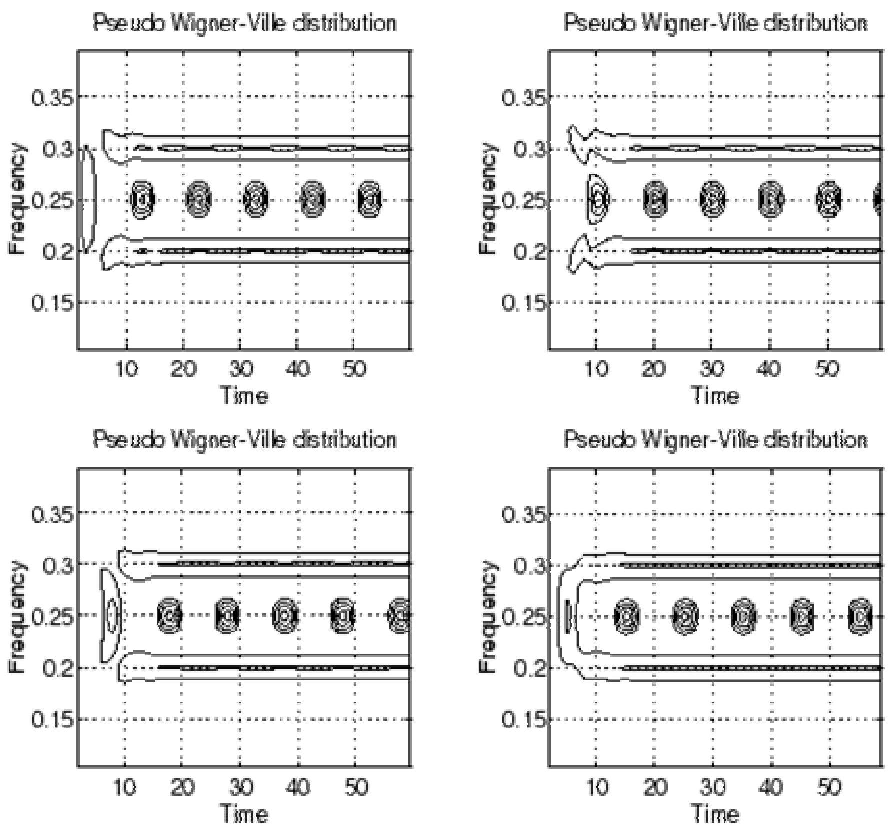

Figure 1. Two simultaneous complex sinusoids analyzed by the Pseudo Wigner-Ville Distribution - the position of the interferences depends on the phase-shift between the two components (π/4, 3π/4, 5π/4 and 7 π/4)

If we compare the Pseudo Wigner-Ville Distribution for different phase shifts, we can observe a time-sliding of the oscillating interferences, like it is presented in figure 1. Each snapshot corresponds to the Pseudo Wigner-Ville Distribution with a different phase shift between the two components.

A second example of signature of the phase is given by the influence of a jump of phase in a signal analyzed by the Pseudo Wigner-Ville Distribution: for instance, if we consider a constant frequency modulation presenting a jump of phase in its middle, like it is presented in figure 2, the Pseudo Wigner-Ville Distribution presents a pattern around the jump position which is all the more important since this jump of phase is close to π. This characteristic can be used to detect a jump of phase in a signal.

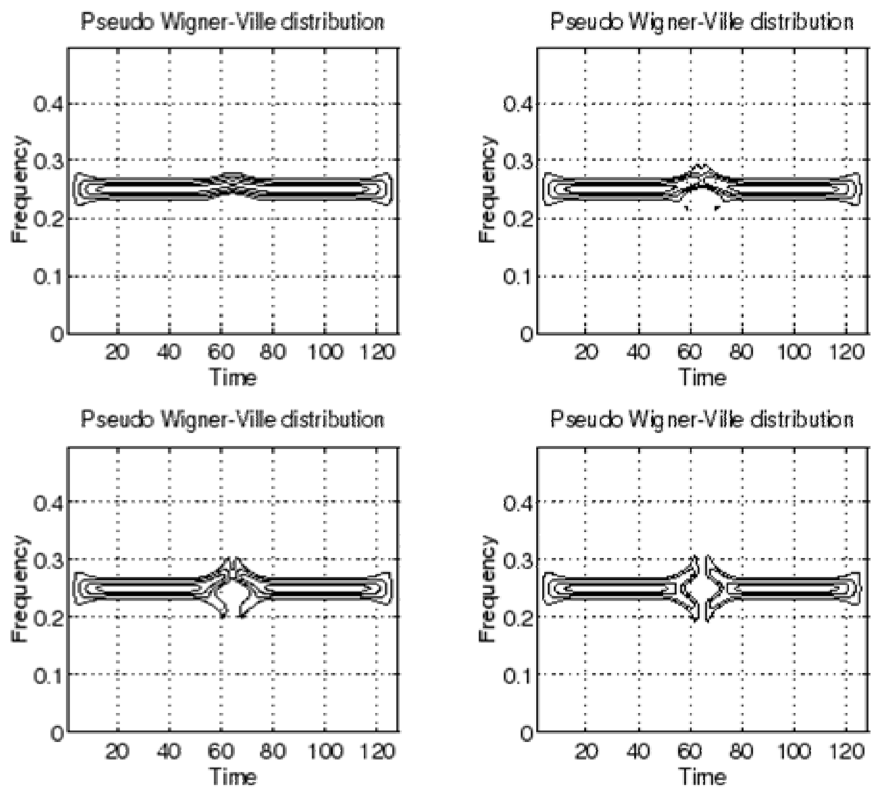

Figure 2. Complex sinusoid presenting a jump of phase in its middle, analyzed by the Pseudo Wigner-Ville Distribution - the shape of the *tfr*-pattern changes with the importance of the jump. These jumps of phase are respectively $\pi/4$, $\pi/2$, $3\pi/4$, π

3. Decomposition of the original signal

Interesting information that one may need to know about an observed non-stationary signal is the number of elementary signals composing this observation.

This also leads us to the following question: how much separation between two elementary signals must one achieve in order to be able to conclude that there are two signals present rather than one ? A solution to this problem is given by applying an information measure to a time-frequency distribution of the signal. Unfortunately, the well known Shannon information, defined as

$$I_x = -\int f(x) \log_2 f(x) dx \tag{8}$$

where *f(x)* is the probability density function of *x*, can not be applied to some time-frequency distributions due to their negative values. The generalized form of information, which admits negative values in the distribution, will then be used. This information, known as *Renyi information*, is given by

$$R_x^{\alpha} = \frac{1}{1-\alpha} \log_2 \left\{ \int_{-\infty}^{+\infty} f^{\alpha}(x) dx \right\} \tag{9}$$

in the continuous case, where α is the order of the information. First order Renyi information ($\alpha = 1$) reduces to Shannon information. Third order Renyi information, applied to a time-frequency distribution *C_x(t,nu),* is defined as

$$R_C^3 = -\frac{1}{2} \log_2 \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} C_x^3(t,\nu) dt d\nu \right\} \tag{10}$$

The result produced by this measure is expressed in *bits*: if one elementary signal yields zero bit of information (2^0), then two well separated elementary signals will yield one bit of information (2^1), four well separated elementary signals will yield two bits of information (2^2), and so on. This can be observed by considering the Wigner-Ville Dostribution of one, two and then four elementary atoms, and then by applying the Renyi information on them.

We can see that if *R* is set to 0 for one elementary atom by subtracting *R1*, we obtain a result close to 1 for two atoms (*R2-R1* = 0.99) and close to 2 for four atoms (R3-R1 = 2.01). If the components are less separated in the time-frequency plane, the information measure will be affected by the overlapping of the components or by the interference terms between them. In particular, it is possible to show that the Renyi information measure provides a good indication of the time separation at which the atoms are essentially resolved, with a better precision than with the time-bandwidth product.

4. Conclusion

The time-frequency representations can be useful tools in decision problems that can have the role of:

- *detect* if an observed signal contains given information (i.e. say, for a given false alarm probability, if *yes* or *no* the information is present);
- *estimate* the parameters of a signal that we know to be present in an observation;
- *classify* a signal in one among different classes.

It has been shown that some of the known optimal strategies of decision can be reformulated equivalently in the time-frequency plane. This result is interesting for two reasons: on one hand, the time-frequency approach, compared to the classical one (formulated in the time-domain in general), usually provides a simpler interpretation of the decision test; on the other hand, when the optimal solution for a given criterion is not known in the decision theory, the time-frequency analysis can be useful to formulate a sub-optimal solution based on the better comprehension of the analyzed signal (for example, a time-frequency detector can be easily modified to take into account variations of the non-stationary signal to be detected, in order to improve the robustness of the detector).

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Adresses:

- Assist. Prof. Dr. Eng. Florin Constantin Berinde, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, f.berinde@uem.ro
- Assoc. Prof. Dr. Eng. Gilbert-Rainer Gillich, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, raini@uem.ro