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Influence of Induction Machine and Mechanism Parameters on Starting Transient Processes in Case of Constant Load Conditions

Two-phase induction machine dynamic model in a coordinate system which rotates at synchronous speed and one-mass dynamic model of mechanism driven in relative units describing transient processes when starting an induction machine in case of constant load conditions are developed.The influence of equivalent circuit parameters of induction machine and mechanism parameters on impact currents and torques and starting time of common used induction machines is studied by means of design of experiment method.

1. Introduction

The modern requirements to the induction electric drives are related to high frequency of switching of the electric motor, impulse modes of operation, frequent reversing, as well as different types of stopping with high precision of establisment. Ignoring the electromechanical transient processes in the analysis of this modes results in wrong evaluation of the torques, acting in the system of the electric drive, significant errors in the definition of the consumed currents, reactive, active and total power, and in calculating the electric motor losses and its heating, and the admissible frequencies of switching, respectively; in inaccuracies in defining the break path of the electric drive, the duration of the transient process, etc.

The induction motor, during the transient processes, develops significant electromagnetic torques, exceeding several times the rated, the starting and even the critical torques. These torques are the reason for the appearance of dangerous mechanical stresses in the elements of the kinematic scheme of the electromechanical system. They cannot be missed in the design, development and evaluation th reliability of operation of the electric drive.

The detail study of the electromagnetic transient processes enables the more rational calculation and design of the induction machine and the mechanical part of the electromechanical system.

The object of this work is to develop dynamic models of the induction machine and the starting mechanism with constant torque. By means of the design of experiment method the influence of the parameters of the substituting scheme and the mechanism will be researched upon the impact currents and torques, and upon the starting time for the general purpose induction motors.

2. Mathematical Model

The research of the transient processes on the induction machines is performed on the base of the generally accepted assumptions [1, 2].

When investigating the transient processes during starting with the adjustable drives it's convenient to work in a synchronously rotating coordinate system x,y, as in this case the sine variables are replaced by constants, and to use as variables both the stator current i_s, and the rotor current i_r [3]. Basing to the summarized mathematical model of the electromechanical transformation of the energy in the induction machines for the above quoted case, after replacement, we obtain the following equation system [3]:

$$\begin{aligned}
\frac{di_{sx}}{dt} &= -\frac{R_s L_r}{L_e} i_{sx} + \left(\omega_s + \frac{L_m^2}{L_e}\omega_r\right) i_{sy} + \frac{R_r L_m}{L_e} i_{rx} + \frac{L_r L_m}{L_e}\omega_r i_{ry} + \frac{L_r}{L_e} u_{sx} \\
\frac{di_{sy}}{dt} &= \left(-\omega_s - \frac{L_m^2}{L_e}\omega_r\right) i_{sx} - \frac{R_s L_r}{L_e} i_{sy} - \frac{L_r L_m}{L_e}\omega_r i_{rx} + \frac{R_r L_m}{L_e} i_{ry} + \frac{L_r}{L_e} u_{sy} \\
\frac{di_{rx}}{dt} &= \frac{R_s L_m}{L_e} i_{sx} - \frac{L_s L_m}{L_e}\omega_r i_{sy} - \frac{R_r L_s}{L_e} i_{rx} + \left(\omega_s + \frac{L_s L_r}{L_e}\omega_r\right) i_{ry} - \frac{L_m}{L_e} u_{sx} \\
\frac{di_{ry}}{dt} &= \frac{L_s L_m}{L_e}\omega_r i_{sx} + \frac{R_s L_m}{L_e} i_{sy} + \left(-\omega_s + \frac{L_s L_r}{L_e}\omega_r\right) i_{rx} - \frac{R_r L_s}{L_e} i_{ry} - \frac{L_m}{L_e} u_{sy}
\end{aligned} \qquad (1)$$

For the transitory value of the electromagnetic torques of the electric motor, while satisfying the requirements for invariability of the power, we obtain:

$$M = z_p L_m \left(i_{sy} i_{rx} - i_{sx} i_{ry}\right)$$

The mechanical part of the electromechanical system most simply can be represented with a single-mass dynamic torque. Then, the motion equation of the electric drive acquires the following expression:

$$\frac{d\omega_r}{dt} = \frac{z_p}{J_\Sigma}\left(M - M_c\right), \qquad (2)$$

where:

M_c - resistant torques of the mechanism;

$J_\Sigma = FI.J_m$ - total inertia moment of the electric motor and the mechanism, reduced to the shaft of the electric motor;

z_p - number of pole pairs of the electric motor;

J_m - inertia moment of the electric motor;

FI - inertia coefficient.

Aiming at convenience during calculations this equation system shall be presented in relative units, using the generally accepted system of basic values [1]. For easier solving the mathematical model is presented in *Cauchy* form, obtaining:

$$\frac{di_{sx}^*}{dt} = \left(-\frac{R_s^* L_r^*}{L_e^*} i_{sx}^* + \left(\omega_s^* + \frac{L_m^{2*}}{L_e^*}\omega_r^* \right) i_{sy}^* + \frac{R_r^* L_m^*}{L_e^*} i_{rx}^* + \frac{L_r^* L_m^*}{L_e^*}\omega_r^* i_{ry}^* + \frac{L_r^*}{L_e^*} u_{sx}^* \right)\omega_b;$$

$$\frac{di_{sy}^*}{dt} = \left(\left(-\omega_s^* - \frac{L_m^{2*}}{L_e^*}\omega_r^* \right) i_{sx}^* - \frac{R_s^* L_r^*}{L_e^*} i_{sy}^* - \frac{L_r^* L_m^*}{L_e^*}\omega_r^* i_{rx}^* + \frac{R_r^* L_m^*}{L_e^*} i_{ry}^* + \frac{L_r^*}{L_e^*} u_{sy}^* \right)\omega_b;$$

$$\frac{di_{rx}^*}{dt} = \left(\frac{R_s^* L_m^*}{L_e^*} i_{sx}^* - \frac{L_s^* L_m^*}{L_e^*}\omega_r^* i_{sy}^* - \frac{R_r^* L_s^*}{L_e^*} i_{rx}^* + \left(\omega_s^* + \frac{L_s^* L_r^*}{L_e^*}\omega_r^* \right) i_{ry}^* - \frac{L_m^*}{L_e^*} u_{sx}^* \right)\omega_b; \quad (3)$$

$$\frac{di_{ry}^*}{dt} = \left(\frac{L_s^* L_m^*}{L_e^*}\omega_r^* i_{sx}^* + \frac{R_s^* L_m^*}{L_e^*} i_{sy}^* + \left(-\omega_s^* + \frac{L_s^* L_r^*}{L_e^*}\omega_r^* \right) i_{rx}^* - \frac{R_r^* L_s^*}{L_e^*} i_{ry}^* - \frac{L_m^*}{L_e^*} u_{sy}^* \right)\omega_b;$$

$$M^* = z_p L_m^* \left(i_{sy}^* i_{rx}^* - i_{sx}^* i_{ry}^* \right);$$

$$\frac{d\omega_r^*}{dt} = \frac{z_p \omega_b}{J_\Sigma^*}\left(M^* - M_c^* \right).$$

For the components of the representation vector of the stator voltages while satisfying the requirements for invariability of the power in relative units we obtain [5]:

$$u_{sx}^* = \sqrt{\frac{2}{3}}\left[u_A^* \cos\omega_s^* t + u_B^* \cos(\omega_s^* t - \frac{2\pi}{3}) + u_C^* \cos(\omega_s^* t + \frac{2\pi}{3}) \right];$$

$$u_{sy}^* = -\sqrt{\frac{2}{3}}\left[u_A^* \sin\omega_s^* t + u_B^* \sin(\omega_s^* t - \frac{2\pi}{3}) + u_C^* \sin(\omega_s^* t + \frac{2\pi}{3}) \right]. \quad (4)$$

$$u_A^* = U_m^* \cos(\omega_s^* t + \varphi_0);$$

$$u_B^* = U_m^* \cos(\omega_s^* t + \varphi_0 - \frac{2\pi}{3}); \quad (5)$$

$$u_C^* = U_m^* \cos(\omega_s^* t + \varphi_0 + \frac{2\pi}{3}),$$

where u_A^*, u_B^* and u_C^* are the phase stator voltages in relative units system.

After replacing (5) in (4) and after transforming, it can be proved that:

$$u^*_{sx} = \sqrt{\frac{2}{3}} U^*_m \frac{3}{2} \cos\varphi_0 = \sqrt{\frac{3}{2}} U^*_m \cos\varphi_0 ; \quad (6)$$

$$u^*_{sy} = -\sqrt{\frac{2}{3}} U^*_m \left(-\frac{3}{2} \right) \sin\varphi_0 = \sqrt{\frac{3}{2}} U^*_m \sin\varphi_0 .$$

where by $U^*_m = U_m / U_б$ the voltage change is taken into account.

φ_0 - initial phase of the supply voltage;

As the electric motors are with short-circuited rotor, $u^*_{rx} = 0$ and $u^*_{ry} = 0$.

To calculate the phase stator currents the following equations are used:

$$i^*_A = \sqrt{\frac{2}{3}} \left(i^*_{sx} \cos\omega^*_s t - i^*_{sy} \sin\omega^*_s t \right)$$

$$i^*_B = \sqrt{\frac{2}{3}} \left(i^*_{sx} \cos\left(\omega^*_s t - \frac{2\pi}{3} \right) - i^*_{sy} \sin\left(\omega^*_s t - \frac{2\pi}{3} \right) \right) \quad (7)$$

$$i^*_C = \sqrt{\frac{2}{3}} \left(i^*_{sx} \cos\left(\omega^*_s t + \frac{2\pi}{3} \right) - i^*_{sy} \sin\left(\omega^*_s t + \frac{2\pi}{3} \right) \right)$$

The reliability and functionality of the presented model has been checked experimentally during previous researches of the authors of two induction electric motors with different powers, as the coincidence between the calculation and experimental results has been very high.

3. Results

When researching a certain electromechanical object it is often observed that some of the interesting qualities, properties or parameters Y depends on some other properties $X_1, X_2, ... X_n$, i.e. there is a function of some variables $Y = f(X_1, X_2, ..., X_n)$. In order to find out more information we define a certain set of influences x, for which we obtain the relevant value of Y. In the most cases the type of the function is not known, therefore the decomposition in degree row is considered.

$$Y = B_0 + B_1 X_1 + ... + B_n X_n + ... + B_{12} X_1 X_2 + ... + $$
$$+ B_{n-1,n} X_{n-1} X_n + B_{11} X_1^2 + ... + B_{nn} X_n^2 \quad (8)$$

Y is called the target function, and the variables $X_1, X_2, ... X_n$, whose influence we investigate, are called factors. When assigning values to the factors, a value for the target function is obtained. In the common cases the factors are quantities with sizes, as their numerical values may considerably differ. Therefore, it is not operated with the actual factor values, but with so called encoded factor values, obtained by means of linear transformation. For the encoding, an output area of the experiment shall be selected, i.e., upper and lower limits must be set for each factor - $X_{i\min}$ and $X_{i\max}$. A new scale is selected for each factor, so that the value of $X_{i\min}$ would correspond to –1, and $X_{i\max}$ to +1.

The expression of Y, obtained in a polynomial form, represents a secondary model of the actual function. The degree of the model obtained may be different, depending on the requirements for simplicity and adequacy of the model. On this base, a second row plan is selected – an orthogonal composition plan (B-plan).

It is necessary, however, to find with the design of experiment method more simple and convenient expressions for the influence of the parameters of the replacing scheme of the induction machine and the mechanism (the factors) upon the impact torque $M_{уд}$, the impact current $i_{уд}$ and the starting time t_s (target functions) of the common induction electric motors at starting for loading with constant torques.

The parameters of the equivalent circuit of the common induction machines in relative units vary within the following limits: $R_s^* = 0{,}01 \div 0{,}08$; $R_r^* = 0{,}02 \div 0{,}08$; $X_{\sigma s}^* = 0{,}06 \div 0{,}14$; $X_{\sigma r}^* = 0{,}06 \div 0{,}16$; $X_m^* = 1{,}2 \div 4{,}0$.

The range of variance of the factors in both actual and encoded form is given in Table 1.

Table 1. Range of variance of the factors in both actual and encoded form

factor	I		II		III		IV		V	
level	R_s^*	x_1	R_r^*	x_2	$X_{\sigma s}^*$	x_3	$X_{\sigma r}^*$	x_4	X_m^*	x_5
lower	0.01	-1	0.02	-1	0.06	-1	0.06	-1	1.2	-1
middle	0.045	0	0.05	0	0.1	0	0.11	0	2.6	0
upper	0.08	+1	0.08	+1	0.14	+1	0.16	+1	4	+1

All possible combinations of the factors can be represented as a table, called plan of experiment. This table contains the meaning or the value of the target function for the relevant combination of the factors and with its help the polynomial coefficients are calculated.

The degree of proximity of the obtained approximating expression to the actual functional dependence of the researched function upon the relevant factors is assessed with the difference between the actual value and the value, as calculated with the approximating expression for a certain experiment. A basic element of the criteria is so called residual sum of the quadratic differences [7]:

$$S = \sum_{u=1}^{N} \left(Y_u - \overline{Y}_u\right)^2$$

By means of this sum, the adequacy dispersion is calculated:

$$\sigma_{a\partial}^2 = \frac{S}{f_{a\partial}}$$

where $f_{a\partial} = N - l$, N is the number of freedom degrees, and l is the number of the coefficients being defined. In the research of electromechanical objects it is convenient to use the following widely applied criterion for the adequacy of the approximating expression obtained:

$$\delta = \sigma_{a\partial} \leq m,$$

where $\sigma_{a\partial}$ is the adequacy dispersion, and m is the admissible value for the accuracy of reproducibility of the function.

For the polynomial model of $M_{y\partial}$, $i_{y\partial}$ and t_s depending on parameters of the replacing scheme for the induction machine, for δ it is obtained $\delta = 0{,}068$, $\delta = 0{,}063$ and $\delta = 0{,}484$, respectively. For the polynomial model of $M_{y\partial}$, $i_{y\partial}$ and t_s depending on parameters of the mechanism, for δ it is obtained $\delta = 3{,}023.10^{-3}$, $\delta = 1{,}286.10^{-3}$ and $\delta = 0{,}175$, respectively.

The mathematical dependencies obtained as a result of the experiment may be considered as an instrument for the research of the dynamics of the induction electric motor. The polynomials $M_{y\partial}, i_{y\partial}, t_s = f(R_s^*, R_r^*, X_{\sigma s}^*, X_{\sigma r}^* X_m^*)$ enables, in particular, the solution of the following tasks [6]:

- to precisely evaluate the influence of the parameters of the dynamic properties;
- to define the "existence zones" for certain values of the target functions;
- to investigate the obtained functional dependencies for the determination of the optimal parameters, ensuring the necessary dynamic characteristics;

The B-plan of second degree confirms the direction and the degree of influence of the factors upon the function being researched, and the coefficients before the quadratic members take into account the non-linearity of the function.

When exploring the influence of the parameters of the equivalent circuit of the induction machine upon $M_{y\partial}$, $i_{y\partial}$ and t_s for the parameters of the mechanisms we assume $M_c^* = 1$ and $FI = 2$, respectively.

For the polynomial model for $M_{y\partial}^*$ we obtain:

$$M_{y\partial}^* = 169{,}506 - 1403{,}001R_s^* - 1767{,}715R_r^* - 992{,}728X_s^* - 914{,}066X_r^* + 3{,}981X_m^* -$$
$$- 841{,}399R_s^*R_r^* + 3760{,}29R_s^*X_s^* + 3266{,}161R_s^*X_r^* - 17{,}165R_s^*X_m^* - 2300{,}755R_r^*X_s^* -$$
$$- 2392{,}771R_r^*X_r^* + 7{,}264R_r^*X_m^* + 2457{,}891X_s^*X_r^* + 2{,}878X_s^*X_m^* - 1{,}335X_r^*X_m^* + \quad (9)$$
$$+ 2344{,}547R_s^{*2} - 6829{,}922R_r^{*2} + 1680{,}356X_s^{*2} - 1546{,}428X_r^{*2} - 0{,}373X_m^{*2}$$

From the analysis of the polynomial model for $M_{y\partial}^*$ it is established, that the increase of R_s^* results in considerable linear decrease of $M_{y\partial}^*$, the increase of R_r^* results in significant nonlinear increase of $M_{y\partial}^*$, the increase of the resistances $X_{\sigma s}^*$ and $X_{\sigma r}^*$ results in considerable approximate linear reduction of $M_{y\partial}^*$, and the increase of X_m^* leads to insignificant nonlinear increase of $M_{y\partial}^*$.

For the polynomial model for $i_{y\partial}^*$ we obtain:

$$i_{y\partial}^* = 20{,}199 - 108{,}971R_s^* - 106{,}179R_r^* - 60{,}254X_s^* - 50{,}082X_r^* - 0{,}285X_m^* + 325{,}06R_s^*R_r^* +$$
$$+ 212{,}098R_s^*X_s^* + 176{,}536R_s^*X_r^* + 0{,}876R_s^*X_m^* + 299{,}01R_r^*X_s^* + 218{,}625R_r^*X_r^* - 0{,}4R_r^*X_m^* +$$
$$+ 84{,}34X_s^*X_r^* + 0{,}66X_s^*X_m^* - 0{,}576X_r^*X_m^* + 178{,}457R_s^{*2} + 145{,}677R_r^{*2} + 48{,}818X_s^{*2} + 40{,}844X_r^{*2} + \quad (10)$$
$$+ 3{,}143.10^{-2}X_m^{*2}$$

The analysis of the polynomial model for $i_{y\partial}^*$ shows that the increase of R_s^* and R_r^* results in considerable approximate linear reduction of $i_{y\partial}^*$, and the increase of $X_{\sigma s}^*$ and $X_{\sigma r}^*$ results in great approximate linear decrease of $i_{y\partial}^*$, and the increase of X_m^* in slight nonlinear increase of $i_{y\partial}^*$.

For the polynomial model for t_s we obtain:

$$t_s = -0{,}13 - 3{,}319R_s^* + 1{,}108R_r^* + 1{,}155X_s^* + 1{,}057X_r^* - 3{,}922.10^{-2}X_m^* + 41{,}071R_s^*R_r^* -$$
$$- 28{,}036R_s^*X_s^* - 27{,}107R_s^*X_r^* - 0{,}898R_s^*X_m^* - 163{,}802R_r^*X_s^* - 152{,}667R_r^*X_r^* -$$
$$- 0{,}118R_r^*X_m^* + 74{,}438X_s^*X_r^* - 0{,}155X_s^*X_m^* - 5{,}714.10^{-2}X_r^*X_m^* + 83{,}839R_s^{*2} + 189{,}115R_r^{*2} + \quad (11)$$
$$+ 30{,}752X_s^{*2} + 26{,}081X_r^{*2} + 2{,}153.10^{-2}X_m^{*2}$$

The analysis of the polynomial model for t_s shows that upon increasing R_s^* the time for starting t_s decreases considerably, reaches the minimum value for $R_s^* \approx 0{,}055$ and after that increase, the increase of R_r^* leads to significant nonlinear decrease of t_s, the increase of $X_{\sigma s}^*$ and $X_{\sigma r}^*$ results in great approximate linear increase of t_s, it also shows that when X_m^* increase t_s slightly decrease, reaches the minimum value at $X_m^* \approx 2{,}5$ and then increases.

The mechanism driven influences on the dynamics of the induction machine through the type and magnitude of the resisting torque, and through the inertia torque of the drive. The dynamics at loading with different constant resistant torques ($M_c^* = 0{,}04 \div 1$) and while changing the inertia coefficient within the range $FI = 1 \div 4$ have been studied. When investigating the influence of the mechanism parameters upon $M_{уд}$, $i_{уд}$ and t_s for the parameters of the equivalent circuit of the induction machine their average values have been applied.

The range of variation of the factors in actual and encoded form is presented in Table 2.

Table 2. Range of variation of the factors in actual and encoded form

factor	I		II	
level	M_c^*	x_1	FI	x_2
lower	0.04	-1	1	-1
middle	0.52	0	2.5	0
upper	1	+1	4	+1

For the polynomial model for $M_{уд}^*$ we obtain:

$$M_{уд}^* = 57{,}94 + 2{,}239M_c^* + 1{,}172FI - 0{,}453M_c^*FI - 0{,}113M_c^{*2} - 0{,}126FI^2 \qquad (12)$$

The analysis of the polynomial model for $M_{уд}^*$ shows that upon increasing M_c^*, $M_{уд}^*$ slightly increases nonlinearly, and that upon increasing FI, $M_{уд}^*$ slightly decrease nonlinearly.

For the polynomial model for $i_{уд}^*$ we obtain:

$$i_{уд}^* = 5{,}498 + 8{,}863.10^{-2}M_c^* + 7{,}153.10^{-3}FI - 1{,}91M_c^*FI + 1{,}085.10^{-3}M_c^{*2} + 3{,}33.10^{-4}FI^2 \qquad (13)$$

The analysis of the polynomial model for $i_{уд}^*$ shows that upon increasing M_c^*, $i_{уд}^*$ slightly increases linearly, and that upon increasing FI, $i_{уд}^*$ slightly decrease nonlinearly.

For the polynomial model for t_s we obtain:

$$t_s = 0{,}17 - 1{,}164M_c^* + 8{,}162.10^{-2}FI + 0{,}339M_c^*FI + 1{,}151M_c^{*2} + 1{,}444.10^{-3}FI^2 \qquad (14)$$

The analysis of the polynomial model for t_s shows that upon increasing M_c^*, t_s increases considerably and nonlinearly, and that upon increasing FI - t_s increases considerably and nonlinearly.

4. Conclusion

Dynamic models of the induction machine and the mechanism for starting with constant torque loading have been developed. Through the design of experiment polynomial models have been obtained to take into account the influence of the parameters of the replacing scheme of the induction machine and the mechanism upon the impact currents and torques, and the time for starting the general purpose induction motors. The reliability and functionality of the models has been checked and the above mentioned dependencies have been investigated. The polynomial models obtained allow for the optimal parameters to be determined, which ensure the necessary dynamic characteristics.

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