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# Geometrical (Degree 0) Modelling of a $FP_3$+3×RTR+$MP_3$ Type Parallel Topology Robotic Guiding Device, Using the „Pair of Frames" (PF) Concept

*The geometrical (degree 0) model of a parallel topology robotic guiding device represents the position-orientation matrix of the mobile platform (MP) versus the fixed one (FP); this model refers to generalized displacements. The kinematical scheme of a $FP_3$+3×RTR+$MP_3$ type mechanism is presented, as well as the manner of choice of the attached pair of frames (PF) to the links. In the case of direct geometrical modelling, for certain displacements of the actuated translational joints, the position-orientation matrix of the mobile platform versus the fixed one is determined. For inverse geometrical modelling, the position-orientation matrix of MP versus FP is known and the displacements of the actuated translational joints are determined.*

## 1. Introduction

A parallel topology robotic guiding device consists, in most of the cases, of a mobile platform (MP) linked with a fixed one (FP) by means of „connexions".

The concepts „connexion" and „pair of frames" (PF) were introduced in Mechanism Theory by Professor F.V. Kovacs. According to [1], [2], a "connexion" is an open linkage interposed between two links, aiming the change of the number of their relative degrees of freedom (DOF).

The concept PF was defined in [4], as well as its using for modelling of mechanical systems. Generalized joints and generalized offsets can be mathematically modelled by means of transformation matrixes between 3D (or 2D) frames, attached in the first case to each of the links constituting the generalized offset [4], [5], [6].

The geometrical model (degree 0) refers to generalized displacements, the kinematical model (degree 1.1) refers to generalized velocities, the kinematical model (degree 1.2) refers to generalized accelerations, the kinetostatical model (degree 2.1) refers to generalized forces and torques in statical conditions and the dynamical model (degree 2.2) refers to generalized forces and torques in dynamical conditions.

## 2. The kinematical scheme and desmodromy of the $FP_3+3\times RTR+MP_3$ type mechanism

The kinematical scheme of a $FP_3+3\times RTR+MP_3$ type parallel topology robotic guiding device mechanism is presented in figure 1:

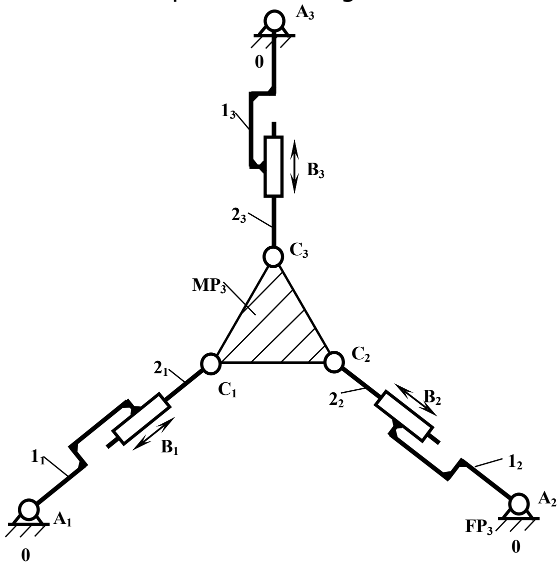


**Figure 1.** The kinematical scheme of a $FP_3+3\times RTR+MP_3$ type mechanism.

Between the fixed platform $FP_3$ and the mobile one $MP_3$ there were interposed 3 identical connexions RTR (every connexion containing two binary links, two rotational joints "R" and a translational one "T"); "j" is the connexion's number (j = 1, 2, 3).

The number of degrees of mobility (DOM) of the mechanism is [3]:

$$M = (6-f)(n_p-1)+\sum_{1}^{3} L_{k\,RTR} - \sum L_p = (6-3)\cdot(2-1)+0-0=3, \qquad (1)$$

where: f - "family" of the kinematical chain (f = 0 for 3D chains and f = 3 for spherical or planar chains); $n_p$ = 2 is the number of the platforms (FP and MP); $L_{k\,RTR}$ - the number of degrees of liberty (DOF) of a connexion; $L_p$ - the number of DOF of the passive linkages resulted by introducing the connexions.

The number of DOF of a RTR connexion, according to [3], is given by:

$$L_{k\,RTR} = (6-f)\cdot n_k - \sum_{i=f+1}^{5}(i-f)\cdot c_{i_k} - \sum L_{id} = (6-3)\cdot 2-(5-3)\cdot 3-0=0\,, \qquad (2)$$

where: $n_k = 2$ is the number of connexion's elements; $c_{ik}$ - the number of the "i" class kinematical pairs of the connexion; $L_{id}$ - the number of useless DOF.

The desmodromy condition is:

$$M = c_{5_c} = 3\,, \qquad (3)$$

where $c_{5_c}$ is the number of the actuated fifth class joints. For the mechanism presented in fig. 1, all the 3 T joints are actuated.

The elements dimensions are given:

- the fixed platform: equilateral triangle having the sides $\overline{A_1A_2} = \overline{A_2A_3} = \overline{A_3A_1} = 200\,[mm]$;
- the mobile platform: equilateral triangle having the sides $\overline{C_1C_2} = \overline{C_2C_3} = \overline{C_3C_1} = 50\,[mm]$;
- link's lenghts $1_j$: $\overline{A_1B_1} = \overline{A_2B_2} = \overline{A_3B_3} = 50\,[mm]$;
- link's lenghts $2_j$: $\overline{B_1C_1} = \overline{B_2C_2} = \overline{B_3C_3} = 70\,[mm]$.

The points $A_j$, $B_j$, $C_j$ were considered in the geometrical centers of the kinematical joints.

### 3. The manner of choice of the frames attached to the connexion's links

In figure 2 is indicated the manner of choice of the frames attached to the links of the connexion $A_jB_jC_j$. The origins of the frames attached by the links of the rotational joints $A_j$ and $C_j$ are placed in the geometrical joints centers $A_j$, $C_j$, respectively in points arbitrary chosen on the platforms: $FP_3$ and $MP_3$, O and P.

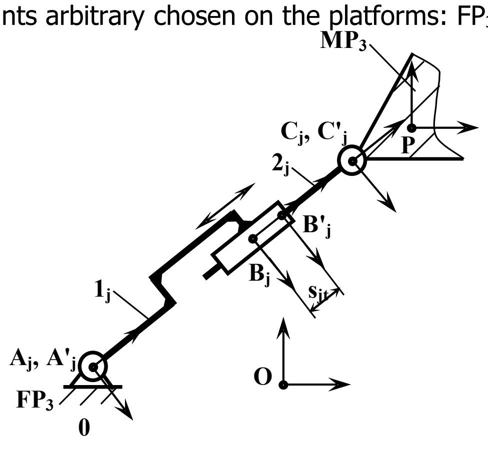


**Figure 2.** The $A_jB_jC_j$ connexion and the attached frames to its links.

The frames will be denominated by means of their origins. The origins of the frames attached to the translational joint $B_j$ will be chosen one on the link $1_j$, $B_j$ and the other on the link $2_j$, $B'_j$, the distance $s_j$ between them, $s_j$ being the relative translational displacement of the two links at the considered time.

$B_jy$ and $B'_jy$ axes will be the same with the translation direction of $B_j$ joint. For simplifying the computation, $A_jy$ and $C_jy$ axes will be chosen on the same direction as the axes $B_jy$ and $B'_jy$; the axes $A_jx$, $A'_jx$, $B_jx$, $B'_jx$, $C_jx$, $C'_jx$ will be chosen parallels.

### 4. The geometrical model of the $FP_3$+3×RTR+$MP_3$ type mechanism

The position-orientation matrix of $MP_3$ versus $FP_3$ is the geometrical model of the connexion. At a certain time t it has the expression:

$$ {}^{FP_3}S_{MP_3} = {}^{O}\underline{T}_{P} = {}^{O}\underline{T}_{A_j} \cdot {}^{A_j}\underline{T}_{A'_j} \cdot {}^{A'_j}\underline{T}_{B_j} \cdot {}^{B_j}\underline{T}_{B'_j} \cdot {}^{B'_j}\underline{T}_{C_j} \cdot {}^{C_j}\underline{T}_{C'_j} \cdot {}^{C'_j}\underline{T}_{P} = \left(\prod_{O}^{P} \underline{T}\right)_j \qquad (4) $$

Because the position-orientation matrix of the characteristic point is the same for every connexion, 3 relations of (4) type can be equalized. Thus, in scalar form, for j = 1, 2, 3, the expression (4) becomes a 3 × 3 × 3 = 27 scalar equation system:

$$ \begin{cases} \left(\prod_{O}^{P} \underline{T}\right)_1 = \left(\prod_{O}^{P} \underline{T}\right)_2 \\ \left(\prod_{O}^{P} \underline{T}\right)_2 = \left(\prod_{O}^{P} \underline{T}\right)_3 \\ \left(\prod_{O}^{P} \underline{T}\right)_1 = \left(\prod_{O}^{P} \underline{T}\right)_3 \end{cases} \qquad (5) $$

If is considered that two by two different position-orientation matrixes are equalized, two times, (for 1 and 2 connexions, 2 and 3 respectively; the equalization of position-orientation matrixes for 1 and 3 connexions leads to redundant equations), and the equalization of the last line of the matrixes leads to redundant equations, the number of equations which can be used to solving the system is 3 × 3 × 2 – 2 × 3 = 12.

For direct geometrical modelling of the parallel topology robotic guiding device, the parameters $s_j$ (j = 1, 2, 3) at the time t imposed by independent external sources of motion and energy will be considered known. By means of the system (5), 6 elements with different values of 0 and 1 of the position - orientation matrix ${}^{FP_3}S_{MP_3}$ and the values of 2 x 3 = 6 variable angles at time t can be computed. That means that 27 – 6 = 21 scalar equations of system (5) are redundant in this case.

For example, for direct geometrical modelling, the parameters $s_j$ (j = 1, 2, 3) at the time t imposed by independent external sources of motion and energy will be considered known:

$$
\begin{aligned}
s_{1t} &= \overline{B_1B'_1} = 10[mm] \\
s_{2t} &= \overline{B_2B'_2} = 10[mm] \qquad (6) \\
s_{3t} &= \overline{B_3B'_3} = 10[mm]
\end{aligned}
$$

The positions and orientations of the attached PF to the connexion’s links are presented in figure 3. The values of $\alpha_j$ şi $\beta_j$ angles are unknown.

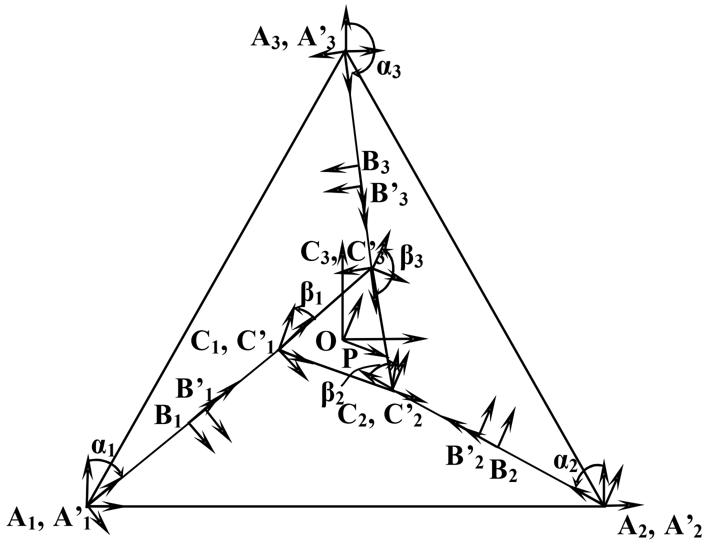


**Figure 3.** Positions and orientations of the attached PF to the mechanism’s elements.

For the connexion 1, j = 1 and relation (4) becomes:

$$
{}^{FP_3}S_{MP_3} = \left({}^{O}\underline{T}_{P}\right)_1 = {}^{O}\underline{T}_{A_1}\cdot{}^{A_1}\underline{T}_{A'_1}\cdot{}^{A'_1}\underline{T}_{B_1}\cdot{}^{B_1}\underline{T}_{B'_1}\cdot{}^{B'_1}\underline{T}_{C_1}\cdot{}^{C_1}\underline{T}_{C'_1}\cdot{}^{C'_1}\underline{T}_{P} = \underline{S}_1, \qquad (7)
$$

where:

$$
{}^{O}\underline{T}_{A_1} = Transl\left[x,\left(l_{OA_1}\right)_x\right]\cdot Transl\left[x,\left(l_{OA_1}\right)_y\right] = \begin{bmatrix} 1 & 0 & \left(l_{OA_1}\right)_x \\ 0 & 1 & \left(l_{OA_1}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & -57,735 \\ 0 & 0 & 1 \end{bmatrix}; \qquad (8)
$$

$$^{A_1}\underline{T}_{A'_1} = Rot(z,\alpha_1) = \begin{bmatrix} cos\alpha_1 & -sin\alpha_1 & 0 \\ sin\alpha_1 & cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\qquad (9)$$

$$^{A'_1}\underline{T}_{B_1} = Transl\left[y,\left(l_{A'_1B_1}\right)_y\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \left(l_{A'_1B_1}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix};\qquad (10)$$

$$^{B_1}\underline{T}_{B'_1} = Transl\left[y,\left(l_{B_1B'_1}\right)_y\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \left(l_{B_1B'_1}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix};\qquad (11)$$

$$^{B'_1}\underline{T}_{C_1} = Transl\left[y,\left(l_{B'_1C_1}\right)_y\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \left(l_{B'_1C_1}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 40 \\ 0 & 0 & 1 \end{bmatrix};\qquad (12)$$

$$^{C_1}\underline{T}_{C'_1} = Rot(y,\beta_1) = \begin{bmatrix} cos\beta_1 & -sin\beta_1 & 0 \\ sin\beta_1 & cos\beta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\qquad (13)$$

$$^{C'_1}\underline{T}_P = Transl\left[y,\left(l_{C'_1P}\right)_y\right] \cdot Transl\left[y,\left(l_{C'_1P}\right)_y\right] = \begin{bmatrix} 1 & 0 & \left(l_{C'_1P}\right)_x \\ 0 & 1 & \left(l_{C'_1P}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 25 \\ 0 & 1 & 14,434 \\ 0 & 0 & 1 \end{bmatrix};\qquad (14)$$

- $\left(l_{OA_j}\right)_x$, $\left(l_{OA_j}\right)_y$ etc., are the projections of the segments $\overline{OA_j}$ etc. on the axes Ox, Oy (for j = 1, 2, 3).

For the connexion j = 2:

$$^{FP_3}S_{MP_3} = \left(^{O}\underline{T}_P\right)_2 = {}^{O}\underline{T}_{A_2} \cdot {}^{A_2}\underline{T}_{A'_2} \cdot {}^{A'_2}\underline{T}_{B_2} \cdot {}^{B_2}\underline{T}_{B'_2} \cdot {}^{B'_2}\underline{T}_{C_2} \cdot {}^{C_2}\underline{T}_{C'_2} \cdot {}^{C'_2}\underline{T}_P = \underline{S}_2,\qquad (15)$$

where:

$$^{O}\underline{T}_{A_2} = \begin{bmatrix} 1 & 0 & \left(l_{OA_2}\right)_x \\ 0 & 1 & \left(l_{OA_2}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & -57,735 \\ 0 & 0 & 1 \end{bmatrix};\qquad (16)$$

$$^{A_2}\underline{T}_{A'_2} = \begin{bmatrix} cos\alpha_2 & -sin\alpha_2 & 0 \\ sin\alpha_2 & cos\alpha_2 & 0 \\ 0 & 0 & 1 \end{bmatrix};\qquad (17)$$

$$ {}^{A'_2}\underline{T}_{B_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (l_{A'_2B_2})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}; \tag{18} $$

$$ {}^{B_2}\underline{T}_{B'_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (l_{B_2B'_2})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}; \tag{19} $$

$$ {}^{B'_2}\underline{T}_{C_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (l_{B'_2C_2})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 40 \\ 0 & 0 & 1 \end{bmatrix}; \tag{20} $$

$$ {}^{C_2}\underline{T}_{C'_2} = \begin{bmatrix} cos\beta_2 & -sin\beta_2 & 0 \\ sin\beta_2 & cos\beta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{21} $$

$$ {}^{C'_2}\underline{T}_{P} = \begin{bmatrix} 1 & 0 & (l_{C'_2P})_x \\ 0 & 1 & (l_{C'_2P})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -25 \\ 0 & 1 & 14{,}434 \\ 0 & 0 & 1 \end{bmatrix}. \tag{22} $$

For the connexion j = 3:

$$ {}^{FP_3}S_{MP_3} = {}^{O}\underline{T}_{P} = {}^{O}\underline{T}_{A_3} \cdot {}^{A_3}\underline{T}_{A'_3} \cdot {}^{A'_3}\underline{T}_{B_3} \cdot {}^{B_3}\underline{T}_{B'_3} \cdot {}^{B'_3}\underline{T}_{C_3} \cdot {}^{C_3}\underline{T}_{C'_3} \cdot {}^{C'_3}\underline{T}_{P} = \underline{S}_3, \tag{23} $$

where:

$$ {}^{O}\underline{T}_{A_3} = \begin{bmatrix} 1 & 0 & (l_{OA_3})_x \\ 0 & 1 & (l_{OA_3})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 115{,}470 \\ 0 & 0 & 1 \end{bmatrix}; \tag{24} $$

$$ {}^{A_3}\underline{T}_{A'_3} = \begin{bmatrix} cos\alpha_3 & -sin\alpha_3 & 0 \\ sin\alpha_3 & cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{25} $$

$$ {}^{A'_3}\underline{T}_{B_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (l_{A'_3B_3})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}; \tag{26} $$

$$ {}^{B_3}\underline{T}_{B'_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (l_{B_3B'_3})_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}; \tag{27} $$

$$ {}^{B'_3}\underline{T}_{C_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \left(l_{B'_3C_3}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 40 \\ 0 & 0 & 1 \end{bmatrix}; \tag{28} $$

$$ {}^{C_3}\underline{T}_{C'_3} = \begin{bmatrix} cos\beta_3 & -sin\beta_3 & 0 \\ sin\beta_3 & cos\beta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{29} $$

$$ {}^{C'_3}\underline{T}_{P} = \begin{bmatrix} 1 & 0 & \left(l_{C'_3P}\right)_x \\ 0 & 1 & \left(l_{C'_3P}\right)_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -28,867 \\ 0 & 0 & 1 \end{bmatrix}. \tag{30} $$

The following equation system is obtained:

$$ \begin{cases} \underline{S}_1 = \underline{S}_2 \\ \underline{S}_2 = \underline{S}_3 \\ sin^2\alpha_1 + cos^2\alpha_1 = 1 \\ sin^2\alpha_2 + cos^2\alpha_2 = 1 \\ sin^2\alpha_3 + cos^2\alpha_3 = 1 \\ sin^2\beta_1 + cos^2\beta_1 = 1 \\ sin^2\beta_2 + cos^2\beta_2 = 1 \\ sin^2\beta_3 + cos^2\beta_3 = 1 \end{cases} \tag{31} $$

Every matrix has 6 elements different of 0 and 1 values, therefore every matrix equation determines 6 scalar equations. In concusion, the (31) system contains 18 equations having 6 unknowns, the angles $\alpha_1$, $\alpha_2$, $\alpha_3$, $\beta_1$, $\beta_2$, $\beta_3$.

The obtained solutions for the equation system are:

$\alpha_1 = -46,9767$ [°]; $\alpha_2 = 73,0233$ [°]; $\alpha_3 = -166,9767$ [°];

$\beta_1 = -4,3411$ [°]; $\beta_2 = -124,3411$ [°]; $\beta_3 = 115,6589$ [°], (32)

and:

$\alpha_1 = -73,0233$ [°]; $\alpha_2 = 46,9767$ [°]; $\alpha_3 = 166,9767$ [°];

$\beta_1 = 124,3411$ [°]; $\beta_2 = 4,3411$ [°]; $\beta_3 = -115,6589$ [°]. (33)

Two position-orientation matrixes are obtained:

$$ {}^{MP_3}S_{FP_3} = \begin{pmatrix} 0,6250 & 0,7806 & 0 \\ -0,7806 & 0,6250 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{34} $$

and

$$
{}^{FP_3}S_{MP_3} = \begin{pmatrix} 0,6250 & -0,7806 & 0 \\ 0,7806 & 0,6250 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (35)
$$

The physical explanation of the existence of two solution sets is: in both of cases the mobile platform $MP_3$ rotates around the characteristic point P, the difference being the rotation sense, as is presented in figure 4. One possible position of $MP_3$ is draw with continuous line, the other one with interrupted line.

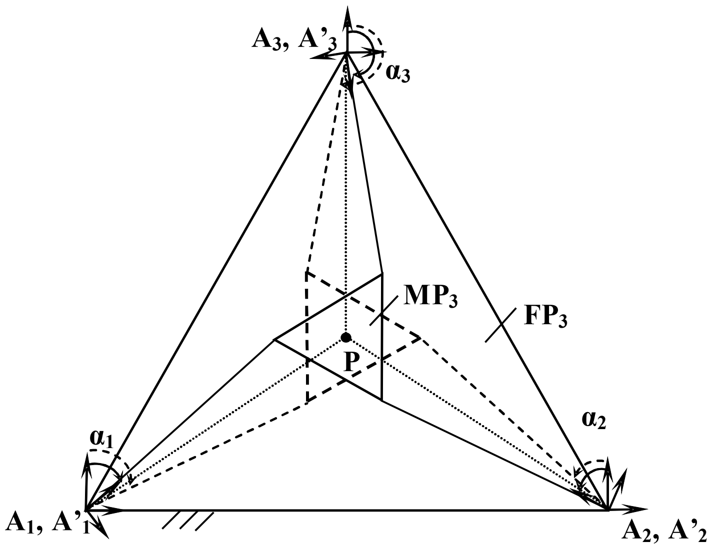


**Figure 4.** Two possible positions of the mobile platform.

For inverse geometrical modelling, the 9 elements of the position - orientation matrix ${}^{FP_3}S_{MP_3}$ at time t are known. The equation system (31) contains 18 scalar equations having 9 unknowns, the $\alpha_1$, $\alpha_2$, $\alpha_3$, $\beta_1$, $\beta_2$, $\beta_3$ angles and the $s_1$, $s_2$, $s_3$ displacements. With the help of the scalar equations of system (31), the values (6) of $s_j$ (j = 1, 2, 3) parameters at time t can be computed, as well as the 6 values (32), (33) of the above mentioned angles, at time t.

**5. Conclusions**

The geometrical modelling of the parallel topology robotic guiding device $FP_3$+3xRTR+$MP_3$ enables the approximative simulation on computer of its functioning.

This modeling can be made for any other mechanism, in the manner presented above.

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